Exercises - 3(b)

1/ Stel

1/ Maximize $z = 5x_1 + 6x_2$

Subject to $2x_1 + 3x_2 \le 6$

$x_1, x_2 \ge 0$

Step-1

Taking the inequation of the constraints as equation i.e

$2x_1 + 3x_2 = 6$

Step-2

Drawing the table and also draw the graph.

$2x_1 + 3x_2 = 6$

x_1	0	3
x_2	2	0

$\Rightarrow 3x_2 = 6 - 2x_1$

$\Rightarrow x_2 = \frac{6-2x_1}{3}$

Step-3

origin (0,0) satisfy the eqn so the shaded region is required feasible region. The vertices of the feasible region is O(0,0)

A (0,2) B (3,0)

Step-4

$z(O) = 5 \cdot 0 + 6 \cdot 0 = 0$

$z(A) = 5 \cdot 0 + 6 \cdot 2 = 12$

$z(B) = 5 \cdot 3 + 6 \cdot 0 = 15$

$\therefore z$ max $= 15$, so the soln of L.P.P is

$x_1 = 3$, $x_2 = 0$

2/ Minimize $z = 6x_1 + 7x_2$

subject to $x_1 + 2x_2 \geq 4$

$x_1, x_2 \geq 0$

Step-1

Taking the inequations of the constraints as equations i.e.

$x_1 + 2x_2 = 4$

Step-2

Drawing the table and also draw the graph:

$\Rightarrow x_1 + 2x_2 = 4$

$\Rightarrow 2x_2 = 4 - x_1$

$\Rightarrow x_2 = \dfrac{4 - x_1}{2}$

x_1	0	2
x_2	2	1

Step-3

origin (0,0) does not satisfy the equation so the shaded region in required feasible region. The vertices of the feasible region is O(0,0) A(0,2) B(2,1)

Step-4

$z(O) = 6 \cdot 0 + 7 \cdot 0 = 0 + 0 = 0$

$z(A) = 6 \cdot 0 + 7 \cdot 2 = 0 + 14 = 14$

$z(B) = 6 \cdot 2 + 7 \cdot 1 = 12 + 7 = 19$

$\therefore$ z min = 14, so the solⁿ of L.P.P is $x_1 = 0$, $x_2 = 2$.

3/ Maximize $z = 20x_1 + 40x_2$

subject to $x_1 + x_2 \le 1$

$6x_1 + 2x_2 \le 3$

$x_1, x_2 \ge 0$

Step-1

Taking the inequation of the constraints as equations i.e.

$x_1 + x_2 = 1$ ——— (i)

$6x_1 + 2x_2 = 3$ ——— (ii)

Step-2

Drawing the table and also draw the graph.

$x_1 + x_2 = 1$

$\Rightarrow x_1 = 1 - x_2$

x_1	0	1
x_2	1	0

$6x_1 + 2x_2 = 3$

$\Rightarrow 2x_2 = 3 - 6x_1$

$\Rightarrow x_2 = \dfrac{3 - 6x_1}{2}$

x_1	0	1
x_2	1.5	-1.5

Step-3

origin (0,0) satisfy the both equation so shaded region in required feasible region. The vertices of the feasible region is $O(0,0)$ $A(0,1)$ $B(1,0)$

In order to get the C point we have to solve both the equation

$$x_1 + x_2 = 1$$
$$6x_1 + 2x_2 = 3$$

$$\text{eqn (1)} \times 6 = \cancel{6x_1} + 6x_2 = 6$$
$$\text{eqn (2)} \times 1 = \cancel{6x_1} + 2x_2 = 3$$
$$\ominus \quad \ominus \quad \ominus$$
$$4x_2 = 3$$
$$x_2 = \frac{3}{4}$$

putting the value of x_2 in eqn (ii)

$$6x_1 + 2x_2 = 3$$
$$\Rightarrow 6x_1 = 3 - 2x_2$$
$$\Rightarrow 6x_1 = 3 - 2 \times \frac{3}{4}$$
$$\Rightarrow 6x_1 = 3 - \frac{6}{4}$$
$$\Rightarrow 6x_1 = \frac{12-6}{4}$$
$$\Rightarrow 6x_1 = \frac{6}{4}$$
$$\Rightarrow x_1 = \frac{\cancel{6}}{4} \times \frac{1}{\cancel{6}}$$
$$\Rightarrow x_1 = \frac{1}{4}$$

So, $C\left(\frac{1}{4}, \frac{3}{4}\right)$

Step-4

$$z(O) = 20 \cdot 0 + 40 \cdot 0 = 0 + 0 = 0$$
$$z(A) = 20 \cdot 0 + 40 \cdot 1 = 0 + 40 = 40$$
$$z(B) = 20 \cdot 1 + 40 \cdot 0 = 20 + 0 = 20$$
$$z(C) = 20 \cdot \frac{1}{4} + 40 \cdot \frac{3}{4} = \frac{20+120}{4} = \frac{140}{4}$$

$\therefore$ $z_{max} = 40$, so the soln of L.P.P is $x_1 = 0$, $x_2 = 1$

4

Minimize $Z = 30x_1 + 45x_2$

Subject to $2x_1 + 6x_2 \geq 4$

$5x_1 + 2x_2 \geq 5$

$x_1, x_2 \geq 0$

Step-1

Taking the inequations of the constraints as equations i.e

$2x_1 + 6x_2 \geq 4$ —— (i)

$5x_1 + 2x_2 \geq 5$ —— (ii)

Step-2

Drawing the table and also draw the graph.

$2x_1 + 6x_2 = 4$

$\Rightarrow 6x_2 = 4 - 2x_1$

$\Rightarrow x_2 = \frac{4 - 2x_1}{6}$

x_1	1	2
x_2	0.3	0

$5x_1 + 2x_2 = 5$

$\Rightarrow 5x_1 = 5 - 2x_2$

$\Rightarrow x_1 = \frac{5 - 2x_2}{5}$

x_1	1	0.2
x_2	0	2

Step-3

origin (0,0) does not satisfy the both eqn so shaded region is required feasible region. The vertices of the feasible region so O (0,0) A (0, 2.5) B( ) C(2,0)

In order to get the B point we have to solve both the eqn:

$$2x_1 + 6x_2 = 4$$
$$5x_1 + 2x_2 = 5$$

$$\text{eqn (1)} \times 1 = 2x_1 + \cancel{6x_2} = 4$$
$$\text{eqn (2)} \times 3 = 15x_1 + \cancel{6x_2} = 15$$
$$(-) \quad (-) \quad (-)$$

$$-13x_1 = -11$$
$$\Rightarrow x_1 = \frac{11}{13}$$

putting the value of x_1 in eqn (1)

$$2x_1 + 6x_2 = 4$$
$$\Rightarrow 6x_2 = 4 - 2x_1$$
$$\Rightarrow 6x_2 = 4 - 2 \times \frac{11}{13}$$
$$\Rightarrow 6x_2 = 4 - \frac{22}{13}$$
$$\Rightarrow 6x_2 = \frac{52 - 22}{13}$$
$$\Rightarrow 6x_2 = \frac{30}{13}$$
$$\Rightarrow x_2 = \frac{\cancel{30}^{5}}{13} \times \frac{1}{\cancel{6}}$$
$$\Rightarrow x_2 = \frac{5}{13}$$

Step-4 So, B $\left(\frac{11}{13}, \frac{5}{13}\right)$

$$Z(O) = 30 \times 0 + 45 \times 0 = 0 + 0 = 0$$
$$Z(A) = 30 \times 0 + 45 \times 2.5 = 112.5$$
$$Z(B) = 30 \times \frac{11}{13} + 45 \times \frac{5}{13} = \frac{330 + 225}{13} = \frac{555}{13}$$
$$Z(C) = 30 \times 2 + 45 \times 0 = 60 + 0 = 60$$

$\therefore Z_{min} = \frac{555}{13}$, so the soln of LPP is

$x_1 = \frac{11}{13}$, $x_2 = \frac{5}{13}$

5/ Maximize $z = 3x_1 + 2x_2$

$-2x_1 + x_2 \le 1$

$x_1 \le 2$

$x_1 + x_2 \le 3$

Step - 1

Taking the inequation of the constraints as equations i.e

$-2x_1 + x_2 = 1$ —— (I)

$x_1 = 2$ —— (II)

$x_1 + x_2 = 3$ —— (III)

Step - 2

Drawing the table and also draw the graph.

$-2x_1 + x_2 = 1$

$x_2 = 1 + 2x_1$

x_1	0	1
x_2	1	3

$x_1 = 2$

x_1	2	2
x_2	0	1

$x_1 + x_2 = 3$

$x_2 = 3 - x_1$

x_1	0	1
x_2	3	2

Step - 3

origin (0,0) satisfy the eqn so the shaded region is required feasible region. The vertices of the feasible region is $O(0,0)$ $B(2,1)$ $C(\; , \;)$

In order to get the c point we have to solve both the eqn

$$-2x_1 + x_2 = 1$$
$$x_1 + x_2 = 3$$

eqn (2)×1

$$x_1 + \cancel{x_2} = 3$$
$$\underset{(-)}{-2x_1} \underset{(-)}{+\cancel{x_2}} = \underset{(-)}{1}$$
$$\overline{\quad 3x_1 = 2 \quad}$$
$$x_1 = \frac{2}{3}$$

Putting the value of x_1 in eqn (1)

$$-2x_1 + x_2 = 1$$
$$\Rightarrow x_2 = 1 + 2x_1$$
$$\Rightarrow x_2 = 1 + 2 \times \frac{2}{3}$$
$$\Rightarrow x_2 = 1 + \frac{4}{3}$$
$$\Rightarrow x_2 = \frac{3+4}{3}$$
$$\Rightarrow x_2 = \frac{7}{3}$$

So, C $\left(\frac{2}{3}, \frac{7}{3}\right)$

Step-4

$$z(O) = 3 \cdot 0 + 2 \cdot 0 = 0 + 0 = 0$$
$$z(A) = 3 \cdot 0 + 2 \cdot 1 = 0 + 2 = 2$$
$$z(B) = 3 \cdot 2 + 2 \cdot 1 = 6 + 2 = 8$$
$$z(C) = 3 \cdot \frac{2}{3} + 2 \cdot \frac{7}{3} = \frac{6+14}{3} = \frac{20}{3}$$

∴ z max = 8, so the soln of L.P.P is $x_1 = 2$, $x_2 = 1$

6/ Maximize $z = 50x_1 + 60x_2$

subject to $x_1 + x_2 \le 5$

$x_1 + 2x_2 \ge 4$

$x_1, x_2 \ge 0$

Step-1

Taking the enequations i.e of the constraints as eqn i.e $x_1 + x_2 = 5$ —— (i)

$x_1 + 2x_2 = 4$ —— (ii)

Step-2 Drawing the table and also draw the graph.

$x_1 + x_2 = 5$

$x_2 = 5 - x_1$

x_1	1	0
x_2	4	5

$x_1 + 2x_2 = 4$

$x_1 = 4 - 2x_2$

x_1	2	4
x_2	1	0

Step-3

origin (0,0) satisfy the eqn (i) but origin does not satisfy the eqn (ii) and (iii) so the feasible region.

shaded region is required feasible region is.

The vertices of the feasible region

O(0,0) (0,5) B (4,0) C ( )

In order to get the C point we have gave to solve both the eqn

$x_1 + x_2 = 5$

$x_1 + 2x_2 = 4$

eqn (i) × 2 $2x_1 + \cancel{2x_2} = 10$
eqn (ii) × 1 $x_1 + \cancel{2x_2} = 4$
$\quad (-) \quad (-) \quad\quad (-)$

$\therefore x_1 = 6$

putting the value of x_1 in eqn (ii)

$x_1 + 2x_2 = 4$
$\Rightarrow 2x_2 = 4 - x_1$
$\Rightarrow 2x_2 = 4 - 1 \times 6$
$\Rightarrow 2x_2 = 4 - 6$
$\Rightarrow 2x_2 = -2$
$\Rightarrow x_2 = \frac{-2}{2} = -1$
$\Rightarrow x_2 = -1$
So, C (6, −1)

Step-4

$Z(O) = 50 \cdot 0 + 60 \cdot 0 = 0 + 0 = 0$
$Z(A) = 50 \cdot 0 + 60 \cdot 5 = 0 + 300 = 300$
$Z(B) = 50 \cdot 4 + 60 \cdot 0 = 200 + 0 = 200$
$Z(C) = 50 \cdot 6 + 60 \cdot -1 = 300 - 60 = 240$

$\therefore Z\ max = 300$ so the soln of L.P.P is $x_1 = 0$, $x_2 = 5$

7/ Maximize $Z = 5x_1 + 7x_2$
subject to $x_1 + x_2 \le 4$
$5x_1 + 8x_2 \le 30$
$10x_1 + 7x_2 \le 35$
$x_1, x_2 \ge 0$

Step-1

Taking the in eqn of constraints as eqn.

$x_1 + x_2 = 4$ —— (i)

$5x_1 + 8x_2 = 30$ —— (ii)

$10x_1 + 7x_2 = 35$ —— (iii)

Step-2

Drawing the table and also draw the graph.

$x_1 + x_2 = 4$

$x_1 = 4 - x_2$

x_1	2	4
x_2	2	0

$5x_1 + 8x_2 = 30$

$5x_1 = 30 - 8x_2$

$x_1 = \frac{30 - 8x_2}{5}$

x_1	6	1.2
x_2	0	3

$10x_1 + 7x_2 = 35$

$10x_1 = 35 - 7x_2$

$x_1 = \frac{35 - 7x_2}{10}$

x_1	0	3.5
x_2	5	0

Step-3

origin (0,0) satisfy the eqn so the shaded region is required feasible region. ~~is~~

The vertices of the feasible region is A(1.2, 3) B (,) C (2, 2) D (3.5, 0)

In order to get the B point we gave solve both the equation

$5x_1 + 8x_2 = 30$

$x_1 + x_2 = 4$

eqn (2) × 1 $\cancel{5x_1} + 8x_2 = 30$

eqn (1) × 5 $\cancel{5x_1} + 5x_2 = 20$

$3x_2 = 10$

$x_2 = \frac{10}{3}$

Putting the value of x_2 in eqn (i)

$$x_1 + x_2 = 4$$

$$\Rightarrow x_1 = 4 - x_2$$

$$\Rightarrow x_1 = 4 - 1 \times \frac{10}{3}$$

$$\Rightarrow x_1 = \frac{12 - 10}{3}$$

$$\Rightarrow x_1 = \frac{2}{3}$$

So, $B\left(\frac{2}{3}, \frac{10}{3}\right)$

Step-4

$Z(O) = 5 \times 0 + 7 \times 0 = 0 + 0 = 0$

$Z(A) = 5 \times \frac{12}{10} + 7 \times 3 = \frac{60}{10} + 21 = 27$

$Z(B) = 5 \times \frac{2}{3} + 7 \times \frac{10}{3} = \frac{10 + 70}{3} = \frac{80}{3}$

$Z(C) = 5 \times 2 + 7 \times 2 = 10 + 14 = 24$

$Z(D) = 5 \times \frac{35}{10} + 7 \times 0 = \frac{175}{10}$

$\therefore Z_{max} = \frac{80}{3}$, so the solution of L.P.P is $x_1 = \frac{2}{3}$, $x_2 = \frac{10}{3}$

8/ Maximize $Z = 14x_1 - 4x_2$

Subject to $x_1 + 2x_2 \le 65$

$7x_1 - 2x_2 \le 25$

$2x_1 + 3x_2 \ge 10$

$x_1, x_2 \ge 0$

Step-1

Taking the ~~inequation~~ of constraints as equation i.e

$x_1 + 2x_2 = 65$ —— (i)

$7x_1 - 2x_2 = 25$ —— (ii)

$2x_1 + 3x_2 = 10$ —— (iii)

Step-2

Drawing the table and also draw the graph.

$x_1+12x_2=65$

$x_1=65-12x_2$

x_1	5	12
x_2	5	4

$7x_1-2x_2=25$

$-2x_2=25-7x_1$

$x_2=\frac{25-7x_1}{-2}$

x_1	3	5
x_2	-2	-5

$2x_1+3x_2=10$

$2x_1=10-3x_2$

$x_1=\frac{10-3x_2}{2}$

x_1	5	2
x_2	0	2

Step-3

origin (0,0) satisfy the eqn (i) and (ii) But the origin does not satisfy eqn (iii) so the shaded region is required feasible region.

The vertices of the feasible region is

A (5, 0) B (2, 2) C (3, 2) D ( )

In order to get the D point we have to both the eqn

$$x_1+12x_2=65$$
$$2x_1+3x_2=10$$

eqn (1) × 1 $\quad x_1+12x_2=65$

eqn (3) × 4 $\quad 8x_1+12x_2=40$

$$-7x_1=25$$
$$x_1=\frac{25}{7}$$

putting the value of x_1 in eqn (ii)

$7x_1-2x_2=25$

$-2x_2=25-7\times\frac{25}{7}$

$=25-\frac{175}{7}$

$=\frac{175-175}{7}$

$=\frac{0}{7}\times\frac{1}{2}$

$=\frac{0}{7}=0$

SO C $\left(\frac{25}{7}, 0\right)$

Step-4

$z(A) = 14\times5 - 4\times0 = 70-0 = 70$

$z(B) = 14\times2 - 4\times2 = 28-8 = 20$

$z(C) = 14\times3 - 4\times2 = 42-8 = 34$

$z(D) = 14\times\frac{25}{7} - 4\times0 = \frac{350}{7} = 50$

$\therefore$ $z_{max} = 50$, so the soln of L.P.P is $x_1 = \frac{25}{7}$, $x_2 = 0$

2/ Maximize $z = 10x_1 + 12x_2 + 8x_3$
subject to $x_1 + 2x_2 \le 30$

$5x_1 - 7x_3 \ge 12$

$x_1 + x_2 + x_3 = 20$

$x_1, x_2 \ge 0$

$x_1 + x_2 + x_3 = 20$

$\Rightarrow x_3 = 20 - (x_1 + x_2)$

$= 20 - x_1 - x_2$ —— (1)

putting the value of x_3 in eqn so that the in eqns become two variables

Maximize $z = 10x_1 + 12x_2 + 8x_3$

$= 10x_1 + 12x_2 + 8(20 - x_1 - x_2)$

Step-1

Taking the inequation of the constraints as equations i.e

$x_1 + 2x_2 = 30$ —— (2)

$5x_1 - 7x_3 = 12$

$5x_1 - 7(20 - x_1 - x_2) = 12$

$\rightarrow 5x_1 - 140 + 7x_1 + 7x_2 = 12$

$\rightarrow 12x_1 + 7x_2 = 12 + 140$

$\rightarrow 12x_1 + 7x_2 = 152$ —— (3)

Step - 2

Drawing the table and also draw the graph.

~~$20 - x_1 - x_2$~~

~~$x_1 + x_2 = 20$~~

~~$x_2 = 20 - x_1$~~

x_1	0	10
x_2	20	10

$x_1 + 2x_2 = 30$

$2x_2 = 30 - x_1$

$x_2 = \frac{30 - x_1}{2}$

x_1	30	10
x_2	0	10

$12x_1 + 7x_2 = 152$

$\Rightarrow x_1 = \frac{152 - 7x_2}{12}$

x_1	1	8
x_2	20	8

Step-3

origin (0,0) satisfy the eqn (1) but origin does not satisfy the eqn (ii) so the shaded region is required feasible region. The vertices of the feasible region is A (30, 0) B (0, 20) C ( )

In order to get the C point we have both the equation

$20 - x_1 - x_2$

$x_1 + x_2 = 20$

$x_1 + 2x_2 = 30$

$eqn(i) \times 2 \quad 2x_1 + 2x_2 = 40$

$eqn(2) \times 1 \quad \underline{x_1 + 2x_2 = 30}$

$x_1 = 10$

putting the value of x_1 in eqn (ii)

$x_1 + 2x_2 = 30$

$\Rightarrow 2x_2 = 30 - x_1$

$\Rightarrow 2x_2 = 30 - 1 \times 10$

$\Rightarrow 2x_2 = 30 - 10$

$\Rightarrow 2x_2 = 20$

$\Rightarrow x_2 = \frac{20}{2}$

$\Rightarrow x_2 = 10$

So, C (10, 10)

Step-4

$Z(A) = 10 \times 30 + 12 \times 0 = 300 + 0 = 300$

$Z(B) = 10 \times 0 + 12 \times 20 = 0 + 240 = 240$

$Z(C) = 10 \times 10 + 12 \times 10 = 100 + 120 = 220$

$\therefore Z \max = 220$

So the solution of L.P.P is

$x_1 = 10, \; x_2 = 10$

10. Minimize $Z = 20x_1 + 10x_2$.

Subject to $x_1 + 2x_2 \le 40$

$3x_1 + x_2 \ge 30$

$4x_1 + 3x_2 \ge 60$

$x_1, x_2 \ge 0$

Step-1

Taking the inequation of constraints as the equation i.e

$x_1 + 2x_2 = 40$ —— (i)

$3x_1 + x_2 = 30$ —— (ii)

$4x_1 + 3x_2 = 60$ —— (iii)

Step-2

Drawing the table and also draw the graph.

$x_1 + 2x_2 = 40$

$x_1 = 40 - 2x_2$

x_1	0	20
x_2	20	10

$3x_1 + x_2 = 30$

$x_2 = 30 - 3x_1$

$x_2 = 30 - 3x_1$

x_1	3	0
x_2	0	30

$4x_1 + 3x_2 = 60$

$4x_1 = 60 - 3x_2$

$x_1 = \frac{60 - 3x_2}{4}$

x_1	0	7.5
x_2	20	10

Step-3

origin (0,0) satisfy the eqn (i) but the origin does not satisfy the eqn (ii) and (iii) so the shaded region is required feasible region. The vertices of the feasible region is.

O(0,0) A(10,0) B(0,20) C(,)

In order to get the c point we have to solve both the eqn

$$3x_1 + x_2 = 30$$
$$4x_1 + 3x_2 = 60$$

eqn (2) × 3 $\quad 9x_1 + 3x_2 = 90$

eqn (3) × 1 $\quad \underline{-4x_1 \mp 3x_2 = -60}$

$$5x_1 = 30$$
$$x_1 = \cancel{\frac{30}{5}}\, 6$$
$$x_1 = 6$$

putting the value of x_1 in eqn (iii)

$$4x_1 + 3x_2 = 60$$
$$3x_2 = 60 - 4 \times 6$$
$$3x_2 = 60 - 24$$
$$x_2 = \cancel{\frac{36}{3}}\, 12$$
$$x_2 = 12$$

So, c (6, 12)

Step-4

$$z(O) = 20 \times 0 + 10 \times 0 = 0 + 0 = 0$$
$$z(A) = 20 \times 10 + 10 \times 0 = 200 + 0 = 200$$
$$z(B) = 20 \times 0 + 10 \times 20 = 0 + 200 = 200.$$
$$z(C) = 20 \times 6 + 10 \times 12 = 120 + 120 = 240$$
$$z(D) = 20 \times 0 + 10 \times 30 = 0 + 30 = 30$$

$\therefore$ z max = 240, so the soln of L.P.P is $x_1 = 6$ $x_2 = 12$

11) Maximize $z = 4x_1 + 3x_2$

subject to $x_1 + x_2 \le 50$

$x_1 + 2x_2 \le 80$

$2x_1 + x_2 \ge 20$

$x_1, x_2 \ge 0$

Step-1

Taking the inequation of the constraints as equation i.e.

$x_1 + x_2 = 50$ —— (i)

$x_1 + 2x_2 = 80$ —— (ii)

$2x_1 + x_2 = 20$ —— (iii)

Step-2

Drawing the table and also draw the graph.

$x_1 + x_2 = 50$

$x_2 = 50 - x_1$

x_1	50	10
x_2	0	40

$x_1 + 2x_2 = 80$

$x_1 = 80 - 2x_2$

x_1	0	20
x_2	40	30

$2x_1 + x_2 = 20$

$x_2 = 20 - 2x_1$

x_1	10	5
x_2	0	10

Step-3

origin (0,0) ~~equation~~ ~~s~~atisfy the (i) and (ii) but origin does not satisfy the eqn (iii) so the shaded region is required feasible region. The vertices of the feasible region

A(50,0) B(0,40) C (x, y)

D(10,0)

In order to get the c point we gave to solve both the eqn.

$x_1 + 2x_2 = 80$
$x_1 + x_2 = 50$

eqn(2)×1 $x_1 + 2x_2 = 80$
eqn(1)×2 $2x_1 + 2x_2 = 100$
(−) (−) (−)

$\Rightarrow -x_1 = \frac{-80}{-1}$

$x_1 = 80$

putting the value of x_1 in eqn (ii)

$x_1 + 2x_2 = 80$
$2x_2 = 80 - x_1$
$\Rightarrow 2x_2 = 80 - 1 \times 80$
$\Rightarrow 2x_2 = 80 - 80$
$\Rightarrow 2x_2 = 0$
$\Rightarrow x_2 = \frac{0}{2}$
$\Rightarrow x_2 = 0$

So c (80, 0)

Step-4

$z(0) = 4 \times 0 + 3 \times 0 = 0 + 0 = 0$
$z(A) = 4 \times 50 + 3 \times 0 = 200 + 0 = 200$
$z(B) = 4 \times 0 + 3 \times 40 = 0 + 120 = 120$
$z(C) = 4 \times 80 + 3 \times 0 = 320 + 0 = 320$
$z(D) = 4 \times 10 + 3 \times 0 = 40 + 0 = 40$

$\therefore$ z max = 900 so the soln of L.P.P is $x_1 = 50$, $x_2 = 0$

14. Optimize $z = -10x_1 + 9x_2$

subject to $-x_1 + x_2 \geq -1$

$x_1 + x_2 \leq 6$

$x_2 \leq 5$

$x_1, x_2 \geq 0$

Step-1

Taking the ~~in~~equations of the constraints as equation i.e

$-x_1 + x_2 = -1$ —— (I)

$x_1 + x_2 = 6$ —— (II)

$x_2 = 5$ —— (III)

Step-2

Drawing the ~~table~~ and also draw the graph.

$-x_1 + x_2 = -1$

$x_2 = -1 + x_1$

x_1	1	3
x_2	0	2

$x_1 + x_2 = 6$

$x_2 = 6 - x_1$

x_1	0	3
x_2	6	3

$x_2 = 5$

x_1	0	4
x_2	5	5

Step-3

origin (0,0) satisfy the eqn (ii) and (iii) but does not satisfy the eqn (i)

So the shaded region. The ventices of the feasible region is A (1, 0) B (   ) C (0, 6) D (0, 5)

In order to get the B point we have to both the eqn

eqn (2) × 1 $\quad x_1 + x_2 = 6$

eqn (1) × 1 $\quad -x_1 + x_2 = -1$

$\quad \overset{(+)}{} \quad \overset{(-)}{} \quad \overset{(+)}{}$

$2x_1 = 7$

$x_1 = \frac{7}{2}$

putting the value x_1 in eqn (1)

$x_1 + x_2 = 6$

$\Rightarrow x_2 = 6 - x_1$

$\Rightarrow x_2 = 6 - 1 \times \frac{7}{2}$

$\Rightarrow x_2 = 6 - \frac{7}{2}$

$\Rightarrow x_2 = \frac{12 - 7}{2}$

$\Rightarrow x_2 = \frac{5}{2}$

So, B $\left(\frac{7}{2}, \frac{5}{2} \right)$

Step-4

~~Z (O) = 4×0 + 3×0 = 0 + 0 = 0~~

Z (A) = -10 × 1 + 2 × 0 = 10 - 0 = 10

$z(B) = -10 \times \frac{7}{2} + 2 \times \frac{5}{2} = \frac{70-10}{2} = \frac{60}{2} = 30$

$z(C) = -10 \times 4 + 2 \times 5 = 40 - 10 = 30$

$z(D) = -10 \times 0 + 2 \times 5 = 0 - 10 = -10$

$\therefore$ $z_{max} = 30$, so the solution of L.P.P is $x_1 = \frac{7}{2}, x_2 = \frac{5}{2}$

$\therefore z_{min} = 10$, so the soln of L.P.P is
$x_1 = 1, x_2 = 0$

12/ Optimize $z = 5x_1 + 25x_2$

Subject to $-0.5x_1 + x_2 \le 2$

$x_1 + x_2 \ge 2$

$-x_1 + 5x_2 \ge 5$

$x_1, x_2 \ge 0$

Step - 1 :

Taking the inequation of the constraints as eqn i.e

$-0.5x_1 + x_2 = 2$ ——— (i)

$x_1 + x_2 = 2$ ——— (ii)

$-x_1 + 5x_2 = 5$ ——— (iii)

Step - 2 Drawing the table and also draw the graph.

$-0.5x_1 + x_2 = 2$

$\Rightarrow x_2 = 2 + 0.5x_1$

x_1	0	2
x_2	2	3

$x_1 + x_2 = 2$

$\Rightarrow x_2 = 2 - x_1$

x_1	1	2
x_2	1	0

$-x_1 + 5x_2 = 5$

$\Rightarrow x_2 = \frac{5 + x_1}{5}$

x_1	0	5
x_2	1	2

Step-3 origin (0,0) satisfy the eqn (i) but origin does not satisfy the eqn (ii) and (iii) so the shaded region is required to feasible region the vartics feasible region is $A=(2,0)$ $B=(0,2)$ $C=?$ $D=(0,1)$

eqn (ii) $\times 1 \Rightarrow x_1 + x_2 = 2$
eqn (iii) $\times 1 \Rightarrow -x_1 + 5x_2 = 5$

$6x_2 = 7$

$x_2 = \frac{7}{6}$

putting the value of x_2 in eqn (ii)

$x_1 + x_2 = 2$

$\Rightarrow x_1 = 2 - \frac{7}{6} = \frac{5}{6}$

So, $C = \left(\frac{5}{6}, \frac{7}{6}\right)$

Step-4

$Z(A) = 5 \times 2 + 25 \times 0 = 10$

$Z(B) = 5 \times 0 + 25 \times 2 = 50$

$Z(C) = 5 \times \frac{5}{6} + 25 \times \frac{7}{6} = \frac{100}{3}$

$\therefore Z_{min} = \frac{100}{3}$ So the soln of L.P.P is $x_1 = \frac{5}{6}$, $x_2 = \frac{7}{6}$

13/

optimize $z = 5x_1 + 2x_2$

subject to $-0.5x_1 + x_2 \le 2$

$x_1 + x_2 \ge 2$

$-x_1 + 5x_2 \ge 5$

$x_1, x_2 \ge 0$

Step-1

Taking the inequation of the constraints as eqn i.e

$-0.5x_1 + x_2 = 2$ —— (1)

$x_1 + x_2 = 2$ —— (11)

$-x_1 + 5x_2 = 5$ —— (111)

Step-2

Drawing the table and also draw the graph.

$-0.5x_1 + x_2 = 2$

$\rightarrow x_2 = 2 + 0.5x_1$

x_1	0	2
x_2	2	3

$x_1 + x_2 = 2$

$\rightarrow x_2 = 2 - x_1$

x_1	1	2
x_2	1	0

$-x_1 + 5x_2 = 5$

$\rightarrow x_2 = \frac{5 + x_1}{5}$

x_1	0	5
x_2	1	2

Step-3

Origin (0,0) satisfy the eqn (i) but origin does not satisfy the eqn (ii) and (iii) so the saded region is required the feasible region. The vartics feasible point is $A=(2,0)$ $B=(0,2)$ $C=?$

In order to get the c point we have solve both the equation.

eqn (ii) $\times 1 \Rightarrow \cancel{x_1}+x_2=2$
eqn (iii) $\times 1 \Rightarrow -\cancel{x_1}+5x_2=5$

$6x_2=7$

$x_2=\frac{7}{6}$

putting the value of x_2 in eqn (ii)

$x_1+x_2=2$

$\Rightarrow x_1=2-\frac{7}{6}=\frac{5}{6}$

Step-4

$z(A)=5\times 2+2\times 0=10$

$z(B)=5\times 0+2\times 2=4$

$z(C)=5\times \frac{5}{6}+2\times \frac{7}{6}=\frac{13}{2}$

$z_{min}=4$, so the L.P.P is $x_1=0, x_2=2$

15/(i) Maximize $z = 1500x_1 + 2000y_1$

subject to $x_1 + 2y_1 \le 24$ ——— (i)

$x_1 + y_1 \le 20$ ——— (ii)

$x_1, y_1 \ge 0$

Step-1

Taking the in eqn of the constraints as eqn i.e.

$x_1 + 2y_1 = 24$

$x_1 + y_1 = 20$

Step-2

Drawing the table and also draw the graph.

$x_1 + 2y_1 = 24$

$\Rightarrow x_1 = 24 - 2y_1$

x_1	0	4
y_1	12	10

$x_1 + y_1 = 20$

$\Rightarrow x_1 = 20 - y_1$

x_1	10	0
y_1	10	20

Step-3

origin (0,0) satisfy the both eqn so the shaded region is required to feasible region. The vertices points are

A=(0,12) B= ? C=(20,0)

In order to get the B points we solve the both eqn.

$$\cancel{x_1} + 2y_1 = 24$$
$$\cancel{x_1} + y_1 = 20$$
$$y_1 = 4$$

Putting the value of y_1 in eqn (ii)

$x_1 + y_1 = 20$

$\Rightarrow x = 20 - 4 = 16$

So, $B = (16, 4)$

Step-4

$Z(A) = 1500 \times 0 + 2000 \times 12 = 24000$

$Z(B) = 1500 \times 16 + 2000 \times 4 = 32000$

$Z(C) = 1500 \times 20 + 2000 \times 0 = 30000$

So, $Z_{max} = 32000$ the soln of L.P.P

is $x_1 = 16, x_2 = 4$

ii) Maximize $z = 15x + 10y$

subject to $2x + 3y \leq 600$

$2x + y \leq 480$

$x, y \geq 0$

Step-1

Taking the in eqn of the constraints as eqn i.e

$2x + 3y = 600$ ——— (i)

$2x + y = 480$ ——— (ii)

Step-2

Drawing the table and also draw the graph

$2x+3y=600$ $\quad$ $2x+y=480$

$\Rightarrow x=\frac{600-3y}{2}$ $\quad$ $\Rightarrow y=480-2x$

x	150	300
y	100	0

x	40	100
y	400	280

Step-3

origin (0,0) satisfy the both eqn so the shaded region required feasible region. The vertics of feasible points are A = (0,200) B = ? C = (240,0)

In order to get the point B we have solve to both the eqn.

$$eqn(1)\times 1 \Rightarrow \cancel{2x}+3y=600$$
$$eqn(11)\times 1 \Rightarrow \cancel{2x}+y=480$$
$$(-)\quad(-)\quad\quad(-)$$
$$2y=120$$
$$\Rightarrow y=\frac{\cancel{120}\,60}{\cancel{2}}$$
$$y=60$$

putting the value of y in eqn (11)

$$2x+y=480$$
$$\Rightarrow 2x=480-60=420$$
$$\Rightarrow x=\frac{\cancel{420}}{\cancel{2}}\,210$$

So, B = (210, 60)

Step-4

$Z(A) = 15\times 0 + 10\times 200 = 2000$

$Z(B) = 15\times 210 + 10\times 60 = 3750$

$Z(C) = 15\times 240 + 10\times 0 = 3600$

$\therefore$ z max = 3750 so the soln of L.P.P is $x_1 = 210$, $x_2 = 60$

iii) Maximize $Z = 20x + 30y$

Subject to $2x + 4y \le 20$

$2x + 2y \le 12$

$4x \le 16$

$x, y \ge 0$

Step-1

Taking the in eqn of the constraints as eqn i.e.

$2x + 4y = 20$ — (i)

$2x + 2y = 12$ — (ii)

$4x = 16$ — (iii)

Step-2

Drawing the table and also draw the graph

$2x + 4y = 20$

$\Rightarrow x = \frac{20 - 4y}{2}$

x	4	0
y	3	5

$2x + 2y = 12$

$\Rightarrow x = \frac{12 - 2y}{2}$

x	6	1
y	0	4

$4x = 16$

$\Rightarrow x = \frac{16}{4} = 4$

x	4	4
y	1	5

Step-3

origin (0,0) satisfy the above eqn so the shaded region required to feasible region. The vertices of feasible points are A(4,0) B(4,2) C(2,4) D(0,5

Step-4

$z(A) = 20 \times 4 + 30 \times 0 = 80$

$z(B) = 20 \times 4 + 30 \times 2 = 140$

$z(C) = 20 \times 2 + 30 \times 4 = 160$

$z(D) = 20 \times 0 + 30 \times 5 = 150$

$\therefore$ z max = 160, so the soln of L.P.P is $x_1 = 2$ $x_2 = 4$

iv) Maximize $z = 15x + 17y$

subject to $4x + 7y \le 150$

$x + y \le 30$

$15x + 17y \ge 300$

$x, y \ge 0$

Step-1

Taking the eqn of the constraints as eqn. i.e

$4x + 7y = 150$ —— (i)

$x + y = 30$ —— (ii)

$15x + 17y = 300$ —— (iii)

Step-2

Drawing the table and also draw the graph.

$4x + 7y = 150$

$\Rightarrow x = \frac{150 - 7y}{4}$

x	35	20
y	0	10

$x + y = 30$

$\Rightarrow x = 30 - y$

x	20	10
y	10	20

$15x + 17y = 300$

$\Rightarrow x = \frac{300 - 17y}{15}$

x	20	5
y	0	10

Step-3

origin (0,0) satisfy the eqn (i) and (ii) but origin does not satisfy the eqn (iii) so the shaded region required to feasible region.

The vertices points are $A=(20,0)$, $B=(30,0)$ $C=(20,10)$ $D=(0,23)$ $E=(0,17)$

Step-4

$Z(A) = 15\times 20 + 17\times 0 = 300$

$Z(B) = 15\times 30 + 17\times 0 = 450$

$Z(C) = 15\times 20 + 17\times 10 = 470$

$Z(D) = 15\times 0 + 17\times 23 = 391$

$Z(E) = 15\times 0 + 17\times 17 = 199$

$\therefore$ $Z_{max} = 470$ the soln of L.P.P is $x_1 = 20$, $x_2 = 10$

v/ Maximize, $Z = 2x + 4y$

subject to $3x + 2y \le 10$ — (i)

$x + 2.5y \le 7.5$ (ii)

$x + 1.2y \le 4.2$

Step-1

Taking the given eqn of the constraints as eqn i.e

$3x + 2y = 10$ — (i)

$x + 2.5y = 7.5$ — (ii)

$x + 1.2y = 4.2$ — (iii)

Step-2

Drawing the table and also draw the graph.

$3x+2y=10$

$\Rightarrow x=\frac{10-2y}{3}$

x	2	0
y	2	5

$x+2.5y=7.5$

$\Rightarrow x=7.5-2.5y$

x	7.5	5
y	0	1

$x+1.2y=4.2$

$\Rightarrow x=4.2-1.2y$

x	4.2	3
y	0	1

Step-3

origin (0,0) satisfy the above eqn so shaded region is required to feasible region. The vartics feasible points are A (3.3,0) B (2.12, 1.8)

C = ? D = (0,3)

In order to get the C point solve the eqn (ii) and (iii)

$$\cancel{x}+2.5y=7.5$$
$$\cancel{x}+1.2y=4.2$$
$$(-)\quad(-)\quad(-)$$
$$1.3y=3.3$$
$$y=\frac{3.3}{1.3}=\frac{33}{\cancel{10}}\times\frac{\cancel{10}}{13}=\frac{33}{13}$$

putting the value of y in eqn (ii)

$x + 2.5y = 7.5$

$\Rightarrow x = \frac{75}{10} - \frac{25}{10} - \frac{25}{10} \times \frac{33}{13} = \frac{15}{2} - \frac{165}{26}$

$\Rightarrow x = \frac{195 - 165}{26} = \frac{30}{26} = \frac{15}{13}$

$\Rightarrow x = \frac{15}{13}$

So, $C = \left(\frac{15}{13}, \frac{33}{13}\right)$

Step-4

$Z(A) = 2 \times 3.3 + 4 \times 0 = \frac{33}{5}$

$Z(B) = 2 \times 2.1 + 4 \times 1.8 = \frac{57}{5}$

$Z(C) = 2 \times \frac{15}{13} + 4 \times \frac{33}{13} = \frac{162}{13}$

$Z(D) = 2 \times 0 + 4 \times 3 = 12$

$\therefore Z_{max} = \frac{162}{13}$, so the soln of L.P.P is $x_1 = \frac{15}{13}$, $x_2 = \frac{33}{13}$

vi) Maximize $Z = 1000x + 800y$

Subject to $x + y \leq 5$

$2x + y \leq 9$

$x, y \geq 0$

Step-1

Taking the eqn of the constraints as eqn i.e

$x + y = 5$ —— (i)

$2x + y = 9$ —— (ii)

Step-2

Drawing the table and also draw the graph.

$x+y=5$

$\Rightarrow x=5-y$

x	4	2
y	1	3

$2x+y=9$

$\Rightarrow y=9-2x$

x	3	2
y	3	5

Step-3

origin $(0,0)$ satisfy the both eqn so the shaded region is required to feasible region. The vartics points are $A(0,5)$ $B(4,1)$ $C(4.5,0)$

Step-4

$z(A) = 1000\times 0 + 800\times 5 = 4000$

$z(B) = 1000\times 4 + 800\times 1 = 4800$

$z(C) = 1000\times 4.5 + 800\times 0 = 4500$

$\therefore z\ max = 4800$ so the soln of L.P.P is $x_1=4,\ x_2=1$

vii) minimize $z = 4960 - 70x - 30y$

subject to $x+y\le 12$

$x \le 8$

$y \le 8$

$x+y \ge 8$

$x, y \ge 0$

Step-1

Taking the in eqn of the constraints as eqn i.e.

$x+y=12$ ——— (1)

$x=8$ ——— (2)

$y=8$ ——— (3)

$x+y=6$ ——— (4)

Step-2

Drawing the table and also draw the graph.

$x+y=12 \Rightarrow x=12-y$

x	8	7
y	4	5

$x=8$

x	8	8
y	2	3

$y=8$

x	6	4
y	8	8

$x+y=6 \Rightarrow x=6-y$

x	5	2
y	1	4

Step-3

origin (0,0) satisfy the eqn (1) (2) and (3) but it is not satisfy the eqn (4) the shaded region is required to feasible region. The vertics points ~~to feasible~~ region ~~The~~ are $A=(6,0)$ $B=(8,0)$ $C=(8,4)$ $D=(4,8)$, $E=(0,6)$

Step-4

$Z(A) = 4960 - 70\times 6 - 130\times 0 = 4540$

$Z(B) = 4960 - 70\times 8 - 130\times 0 = 4400$

$Z(C) = 4960 - 70\times 8 - 130\times 4 = 4384$

$Z(D) = 4960 - 70\times 4 - 130\times 8 = 3640$

$Z(E) = 4960 - 70\times 0 - 130\times 6 = 4180$

$\therefore Z_{min} = 3640$ so the soln L.P.P is

$x_1 = 4, x_2 = 8$

viii) Minimize $z = 18x + 20y$

subject to $x + 2y \geq 10$

$x + y \geq 6$

$3x + y \geq 8$

$x, y \geq 0$

Step-1

Taking the in eqn of the constraints as eqn i.e

$x + 2y = 10$

$x + y = 6$

$3x + y = 8$

Step-2

Drawing the table and also draw the graph.

$x + 2y = 10$

$\Rightarrow x = 10 - 2y$

x	0	6
y	5	2

$x + y = 6$

$\Rightarrow x = 6 - y$

x	3	4
y	3	2

$3x + y = 8$

$\Rightarrow y = 8 - 3x$

x	1	2
y	5	2

Step-3

origin (0,0) does not satisfy the above eqn so the shaded region is required to feasible region. The vertices of feasible points are A = (2,4), B = (1,5).

Step-4

$z(A) = 16 \times 2 + 20 \times 4 = 112$

$z(B) = 16 \times 1 + 20 \times 5 = 116$

$\therefore z_{min} = 112$, so the solⁿ of L.P.P is $x_1 = 2, x_2 = 4$

i.e. minimize $z = (512.5)x + 800y$

Subject to : $5x + 4y = 40$

$x \leq 7$

$y \leq 3$

$x, y \geq 0$

Step-1

Taking the eqⁿ of the constraints as eqⁿ i.e.

$5x + 4y = 40$ —— (i)

$x = 7$ —— (ii)

$y = 3$ —— (iii)

Step-2

Drawing the table and also draw the graph.

$5x + 4y = 40$

$\Rightarrow x = \frac{40 - 4y}{5}$

x	8	4
y	0	5

$x = 7$

x	7	7
y	1	4

$y = 3$

x	5	6
y	3	3

Step-3

origin (0,0) satisfy the eqⁿ so the shaded region is required to feasible region. The vantics of feasible points are A(7,0), B(7,1.2) & C C=? D= (0,3)

In order to get c points we solve the above eqn.

eqn(i) $\times$ 1 $\Rightarrow$ $5x+4y=40$

eqn (iii) $\times$ 4 $\Rightarrow$ $\underline{(-)\ 4y=12}$

$5x=28$

$x=\frac{28}{5}$

putting the value of x in eqn(i)

$5x+4y=40$

$\Rightarrow 4y=40-28=12$

$\Rightarrow y=\frac{12}{4}=3$

So, $C=\left(\frac{28}{5},3\right)$

Step-4

$z(A)=512.5\times7+800\times0=3587$

$z(B)=512.5\times7+800\times1.2=\frac{9095}{2}$

$z(C)=512.5\times\frac{28}{5}+800\times3=5270$

$z(D)=512.5\times0+800\times3=2400$

$\therefore$ z min = 5270, the soln of L.p.p is

$x_1=\frac{28}{5}, x_2=3$