Exercise - 4 (b)

Date - 29.08.23

1/ (i) Symmetric

ii/ Neither symmetric nor skew symmetric

iii/ symmetric

iv/ skew symmetric

v/ Both

vi/ ~~Ne~~ Niether

vii/ skew symmetric

2/ i/ T ii/ T iii/ F iv/ T v/ F vi/ T vii/ F

3/ i/ A and B are symmetric matrics

Thus $A' = A$ and $B' = B$

Now, $(AB - BA)' = (AB)' - (BA)'$

$= B'A' - A'B'$

$= BA - AB$

$= -(AB - BA)$

$\therefore$ $AB - BA$ is skew symmetric

ii/ We know that zero matrix is both symmetric as well as skew symmetric.

Let A is symmetric.

$\therefore$ $A = A + 0$ where A is symmetric and 0

is treated as skew symmetric
If B is skew symmetric then we can write
$B = O + B$ where O is symmetric and B is skew symmetric.

4/ (i) Let A, B and AB are all symmetric

$\therefore A' = A, B' = B$ and

$(AB)' = AB$

$\Rightarrow B'A' = AB$

$\Rightarrow BA = AB$

$\Rightarrow AB - BA = 0$

(ii) Let A, B and AB are all skew symmetric matrices

$A' = -A$, $B' = -B$ and $(AB)' = -AB$

Now $(AB)' = -AB$

$\Rightarrow B'A' = -AB$

$\Rightarrow (-B)(-A) = -AB$

$\Rightarrow BA = -AB$

$\Rightarrow BA + AB = 0$

5/ If $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 2 & 3 \\ -2 & 5 & 3 \end{bmatrix}$ then verify that

$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ -2 & 5 & 3 \end{bmatrix}$

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$$A' = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 1 & -5 \\ 0 & 3 & 3 \end{bmatrix}$$

i) $A+A'$ is symmetric

Ans→ Now $P = A + A' = \begin{bmatrix} 2 & 2 & -2 \\ 2 & 2 & 8 \\ -2 & 8 & 6 \end{bmatrix}$

$$P' = \begin{bmatrix} 2 & 2 & -2 \\ 2 & 2 & 8 \\ -2 & 8 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & -2 \\ 2 & 2 & 8 \\ -2 & 8 & 6 \end{bmatrix} = P$$

$\therefore$ $A+A'$ is symmetric

ii) $A-A'$ is skew symmetric

Ans→ $Q = A - A' = \begin{bmatrix} 0 & 2 & 2 \\ -2 & 0 & -2 \\ -2 & 2 & 0 \end{bmatrix}$

$$Q' = \begin{bmatrix} 0 & -2 & -2 \\ 2 & 0 & 2 \\ 2 & -2 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 2 & 2 \\ -2 & 0 & -2 \\ -2 & 2 & 0 \end{bmatrix}$$

$$= -Q$$

$\therefore$ $A-A'$ is skew symmetric

6) If $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$ show that $A^2 = I$ and hence $A = A^{-1}$

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Ans => No the converse is not true for example:

$$A = \begin{bmatrix} 4 & -1 \\ 15 & -4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -4 & 1 \\ -15 & -4 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 15 & -4 \end{bmatrix} = A$$

7/ $$A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} \cancel{0\times 0 + 1\times 4 + (-1)\times 3} \\ \cancel{4\times 1 + -3\times -3 + 4\times -3} \\ \cancel{3\times -1 + -3\times 4 + 4\times 4} \end{bmatrix}$$

$$= \cancel{0\times 0 + 1\times 4 + (-1)\times 3} \quad \cancel{0\times 1 + 1\times(-3) + (-1)\times}$$

$$= \cancel{0\times 0 + 1\times 4 + -1\times 3}$$

$$= \begin{bmatrix} 0\times 0 + 1\times 4 + (-1)\times 3 & 0\times 1 + 1\times(-3) + (-1)\times(-3) & 0\times(-1) + 1\times 4 + -1\times 4 \\ 4\times 0 + (-3)\times 4 + 4\times 3 & 4\times 1 + (-3)\times(-3) + 4\times(-3) & 4\times(-1) + (-3)\times 4 + 4\times 4 \\ 3\times 0 + -3\times 4 + 4\times 3 & 3\times 1 + -3\times -3 + 4\times(-3) & 3\times -1 + -3\times 4 + 4\times 4 \end{bmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$$

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$\Rightarrow A^{-1} A^2 = A^{-1} I$

Q/ Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$|A| = 0 - 1$
$= -1$ since $-1 \neq 0$

$A_{11} = (-1)^{1+1} \times 0 = 0$

$A_{12} = (-1)^{1+2} \times 1 = -1$

$A_{21} = (-1)^{2+1} \times 1 = -1$

$A_{22} = (-1)^{2+1} \times 0 = 0$

$Adj\,A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

$A^{-1} = \frac{Adj\,A}{|A|} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = A$ (proved)

Date: 31.08.23

Q/ Let $I^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ be the unit matrix

$I \cdot I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow I^{-1} = I$

$AB = I$
$A^{-1} = B$
$B^{-1} = A$
$A \cdot A = I$
$A^2 = I$
$A^{-1} = A$

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Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$A \cdot A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= I$$

$\Rightarrow A = A^{-1}$ (the converse statement is false)

7/ $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$

$$A^2 = A \cdot A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= I$$

$$\therefore A = A^{-1}$$

9/ i/ $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$I^2 = I \cdot I$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \Rightarrow I = I^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

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ii) $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore I^2 = I$

$\therefore I^{-1} = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2. i) $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 1 \\ -1 & 5 & -2 \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 1 \\ -1 & 5 & -2 \end{bmatrix}^T$

$= \begin{bmatrix} 1 & 4 & -1 \\ 2 & 0 & 5 \\ 3 & 1 & -2 \end{bmatrix}$

$A + A^T = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 1 \\ -1 & 5 & -2 \end{bmatrix} + \begin{bmatrix} 1 & 4 & -1 \\ 2 & 0 & 5 \\ 3 & 1 & -2 \end{bmatrix}$

$= \begin{bmatrix} 2 & 6 & 2 \\ 6 & 0 & 6 \\ 2 & 6 & -4 \end{bmatrix}$

$A - A^T = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 1 \\ -1 & 5 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 4 & -1 \\ 2 & 0 & 5 \\ 3 & 1 & -2 \end{bmatrix}$

$= \begin{bmatrix} 0 & -2 & 4 \\ 2 & 0 & -4 \\ -4 & 4 & 0 \end{bmatrix}$

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we have

$$A = \frac{1}{2}(A+A^T) + \frac{1}{2}(A-A^T)$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 1 \\ -1 & 5 & -2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 & 6 & 2 \\ 6 & 0 & 6 \\ 2 & 6 & -4 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & -2 & 4 \\ 2 & 0 & -4 \\ -4 & 4 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 1 \\ -1 & 5 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 1 \\ 3 & 0 & 3 \\ 1 & 3 & -2 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -2 \\ -2 & 2 & 0 \end{bmatrix}$$

symmetric + skewsymmetric

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ii) $\begin{bmatrix} 2 & -1 & 3 \\ 5 & 7 & -2 \\ 1 & 4 & 6 \end{bmatrix}$

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 5 & 7 & -2 \\ 1 & 4 & 6 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 2 & -1 & 3 \\ 5 & 7 & -2 \\ 1 & 4 & 6 \end{bmatrix}^T$$

$$= \begin{bmatrix} 2 & 5 & 1 \\ -1 & 7 & 4 \\ 3 & -2 & 6 \end{bmatrix}$$

$$A+A^T=\begin{bmatrix}2&-1&3\\5&7&-2\\1&4&6\end{bmatrix}+\begin{bmatrix}2&5&1\\-1&7&4\\3&-2&6\end{bmatrix}$$

$$=\begin{bmatrix}2+2&-1+5&3+1\\5+(-1)&7+7&-2+4\\1+3&4+(-2)&6+6\end{bmatrix}$$

$$=\begin{bmatrix}4&4&4\\4&14&2\\4&2&12\end{bmatrix}$$

$$A-A^T=\begin{bmatrix}2&-1&3\\5&7&-2\\1&4&6\end{bmatrix}-\begin{bmatrix}2&5&1\\-1&7&4\\3&-2&6\end{bmatrix}$$

$$=\begin{bmatrix}2-2&-1-5&3-1\\5-(-1)&7-7&-2-4\\1-3&4-(-2)&6-6\end{bmatrix}=\begin{bmatrix}0&-6&2\\4&0&-6\\-2&6&0\end{bmatrix}$$

we have

$$A=\frac{1}{2}(A+A^T)+\frac{1}{2}(A-A^T)$$

$$\begin{bmatrix}2&-1&3\\5&7&-2\\1&4&6\end{bmatrix}=\frac{1}{2}\begin{bmatrix}4&4&4\\4&14&2\\4&2&12\end{bmatrix}+\frac{1}{2}\begin{bmatrix}0&-6&2\\4&0&-6\\-2&6&0\end{bmatrix}$$

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$$\begin{bmatrix} 2 & -1 & 3 \\ 5 & 7 & -2 \\ 1 & 4 & 6 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 7 & 1 \\ 2 & 1 & 6 \end{bmatrix} + \begin{bmatrix} 0 & -3 & 1 \\ 3 & 0 & -3 \\ -1 & 3 & 0 \end{bmatrix}$$

symmetric + skew symmetric

iii)

$$A = \begin{bmatrix} x & a & b \\ a & y & c \\ b & c & z \end{bmatrix}$$

$$A^T = \begin{bmatrix} x & a & b \\ a & y & c \\ b & c & z \end{bmatrix}^T$$

$$= \begin{bmatrix} x & a & b \\ a & y & c \\ b & c & z \end{bmatrix}$$

$$A + A^T = \begin{bmatrix} x & a & b \\ a & y & c \\ b & c & z \end{bmatrix} + \begin{bmatrix} x & a & b \\ a & y & c \\ b & c & z \end{bmatrix}$$

$$= \begin{bmatrix} x+x & a+a & b+b \\ a+a & y+y & c+c \\ b+b & c+c & z+z \end{bmatrix}$$

$$= \begin{bmatrix} 2x & 2a & 2b \\ 2a & 2y & 2c \\ 2b & 2c & 2z \end{bmatrix}$$

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$$A - A^T = \begin{bmatrix} x & a & b \\ a & y & c \\ b & c & z \end{bmatrix} - \begin{bmatrix} x & a & b \\ a & y & c \\ b & c & z \end{bmatrix}$$

$$= \begin{bmatrix} x-x & a-a & b-b \\ a-a & y-y & c-c \\ b-b & c-c & z-z \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

we have

$$A = \frac{1}{2}(A+A^T) + \frac{1}{2}(A-A^T)$$

$$= \begin{bmatrix} x & a & b \\ a & y & c \\ b & c & z \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2x & 2a & 2b \\ 2a & 2y & 2c \\ 2b & 2c & 2z \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} x & a & b \\ a & y & c \\ b & c & z \end{bmatrix} = \begin{bmatrix} x & a & b \\ a & y & c \\ b & c & z \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

symmetric + skew symmetric

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iv $\begin{bmatrix} 0 & x \\ -x & 0 \end{bmatrix}$

$A = \begin{bmatrix} 0 & x \\ -x & 0 \end{bmatrix}$

$A^T = \begin{bmatrix} 0 & x \\ -x & 0 \end{bmatrix}^T$

$= \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}$

$A + A^T = \begin{bmatrix} 0 & x \\ -x & 0 \end{bmatrix} + \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}$

$= \begin{bmatrix} 0+0 & x+(-x) \\ -x+x & 0+0 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$A - A^T = \begin{bmatrix} 0 & x \\ -x & 0 \end{bmatrix} - \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}$

$= \begin{bmatrix} 0-0 & x+x \\ -x-x & 0-0 \end{bmatrix}$

$= \begin{bmatrix} 0 & 2x \\ -2x & 0 \end{bmatrix}$

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we have

$$A = \frac{1}{2}(A + AT) + \frac{1}{2}(A - AT)$$

$$\begin{bmatrix} 0 & x \\ -x & 0 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 2x \\ -2x & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & x \\ -x & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & x \\ -x & 0 \end{bmatrix}$$

skew symmetric + symmetric

v/ $A = \begin{bmatrix} 1 & 5 \\ 7 & -3 \end{bmatrix}$

$$AT = \begin{bmatrix} 1 & 5 \\ 7 & -3 \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & 7 \\ 5 & -3 \end{bmatrix}$$

$$A + AT = \begin{bmatrix} 1 & 5 \\ 7 & -3 \end{bmatrix} + \begin{bmatrix} 1 & 7 \\ 5 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 5+7 \\ 7+5 & -3+(-3) \end{bmatrix} = \begin{bmatrix} 2 & 12 \\ 12 & -6 \end{bmatrix}$$

$$A - AT = \begin{bmatrix} 1 & 5 \\ 7 & -3 \end{bmatrix} - \begin{bmatrix} 1 & 7 \\ 5 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 5-7 \\ 7-5 & -3+3 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$$

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we have

$$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$$

$$\begin{bmatrix} 1 & 5 \\ 7 & -3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 & 12 \\ 12 & -6 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 5 \\ 7 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ 6 & -3 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

symmetric + skew symmetric

ii)
$$A = \begin{bmatrix} 4 & -3 \\ 1 & 2 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 4 & -3 \\ 1 & 2 \end{bmatrix}^T$$

$$= \begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$$

$$A + A^T = \begin{bmatrix} 4 & -3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+4 & -3+1 \\ 1+(-3) & 2+2 \end{bmatrix} = \begin{bmatrix} 8 & -2 \\ -2 & 4 \end{bmatrix}$$

$$A - A^T = \begin{bmatrix} 4 & -3 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4-4 & -3-1 \\ 1-(-3) & 2-2 \end{bmatrix} = \begin{bmatrix} 0 & -4 \\ 4 & 0 \end{bmatrix}$$

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we have

$$A = \frac{1}{2}(A + AT) + \frac{1}{2}(A - AT)$$

$$\begin{bmatrix} 4 & -3 \\ 1 & 2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 8 & -2 \\ -2 & 4 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & -4 \\ 4 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$$

symmetric + skew symmetric

vii)
$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$AT = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A + AT = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$

$$A - AT = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

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we have

$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

symmetric + skew symmetric

Date 02.09.23

11/i

$A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$

Let $A = IA$

$\Rightarrow \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A \quad [R_2 \to R_2 - 3R_1]$

$\Rightarrow \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} A \quad [R_1 \to R_1 + 2R_2]$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -5 & 2 \\ -3 & 1 \end{bmatrix} A \quad [R_2 \to -R_2]$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix} A$

$\therefore A^{-1} = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$

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ii) $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

Let $A = IA$

$\Rightarrow \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A \quad [R_2 \to 2R_2]$

$\Rightarrow \begin{bmatrix} 2 & 5 \\ 2 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A \quad [R_2 \to R_2 - R_1]$

$\Rightarrow \begin{bmatrix} 2 & 5 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} A \quad [R_1 \to R_1 - 5R_2]$

$\Rightarrow \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 & -10 \\ -1 & 2 \end{bmatrix} A \quad [R_1 \to \frac{1}{2} R_1]$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} A$

$\therefore A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

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iii) $A = \begin{bmatrix} 4 & -2 \\ 3 & 1 \end{bmatrix}$

Let $A = IA$

$\Rightarrow \begin{bmatrix} 4 & -2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A \quad [R_2 \to 4R_2]$

$\Rightarrow \begin{bmatrix} 4 & -2 \\ 12 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} A \quad [R_2 \to R_2 - 3R_1]$

$\Rightarrow \begin{bmatrix} 4 & -2 \\ 0 & 10 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ -3 & 4 \end{bmatrix} A \quad [R_1 \to 5R_1]$

$$\Rightarrow \begin{bmatrix} 20 & -10 \\ 0 & 10 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ -3 & 4 \end{bmatrix} A \quad [R_1 \to R_1 + R_2]$$

$$\Rightarrow \begin{bmatrix} 20 & 0 \\ 0 & 10 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ -3 & 4 \end{bmatrix} A \quad [R_1 \to \tfrac{1}{10} R_1, R_2 \to \tfrac{1}{10} R_2]$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix} A$$

$$\therefore A^{-1} = \frac{1}{10} \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} \frac{1}{10} & \frac{1}{5} \\ \frac{-3}{10} & \frac{2}{5} \end{bmatrix}$$

iv) $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

Let $A = IA$

$$\Rightarrow \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A \quad [R_2 \to 2R_2]$$

$$\Rightarrow \begin{bmatrix} 2 & 5 \\ 2 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} A \quad [R_2 \to R_2 - R_1]$$

$$\Rightarrow \begin{bmatrix} 2 & 5 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} A \quad [R_1 \to R_1 - 5R_2]$$

$$\Rightarrow \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 & -10 \\ -1 & 2 \end{bmatrix} A \quad (R_1 \to \tfrac{1}{2} R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

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v) $A = \begin{bmatrix} 1 & 0 \\ 2 & -3 \end{bmatrix}$

Let $A = IA$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A \quad [R_2 \to R_2 - 2R_1]$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A \quad [R_2 \to -\frac{1}{3} R_2]$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{2}{3} & -\frac{1}{3} \end{bmatrix} A$

$\Rightarrow A^{-1} = \begin{bmatrix} 1 & 0 \\ \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$

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vi) $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

Let $A = IA$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A \quad [R_2 \to -1R_2]$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} A$

$\therefore A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

12) i) $A = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix}$

Let $A = IA$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad [R_1 \rightarrow R_3]$$

$$\Rightarrow \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A \quad \left[R_1 \rightarrow \tfrac{1}{2}R_1, R_2 \rightarrow \tfrac{1}{2}R_2, R_3 \rightarrow \tfrac{1}{2}R_3\right]$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A$$

$$\therefore A^{-1} = \frac{1}{2} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 \end{bmatrix}$$

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ii/ $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

Let $A = IA$

$$\Rightarrow \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad [R_1 \leftrightarrow R_2]$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad [R_3 \rightarrow R_3 - 3R_1]$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} A \quad [R_1 \rightarrow R_1 - 2R_2, R_3 \rightarrow R_3 + 5R_2]$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 2 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix} A \quad [R_1 \to 2R_1,\ R_2 \to R_2 - R_3]$$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -4 & 2 & 0 \\ -4 & 3 & -1 \\ 5 & -3 & 1 \end{bmatrix} A \quad [R_1 \to R_1 + R_3]$$

$$\Rightarrow \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -4 & 3 & -1 \\ 5 & -3 & 1 \end{bmatrix} A \quad [R_1 \to \tfrac{1}{2}R_1,\ R_3 \to \tfrac{1}{2}R_3]$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & -1 \\ 5/2 & -3/2 & 1/2 \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & -1 \\ 5/2 & -3/2 & 1/2 \end{bmatrix}$$

ii) $A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$

Let $A = IA$

$$\Rightarrow \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad [R_2 \to 3R_2,\ R_3 \to 3R_3]$$

$\Rightarrow$ 3 −2 3
2 7

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$$\Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 2 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix} A \quad [R_1 \to 2R_1,\ R_2 \to R_2 - R_3]$$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -4 & 2 & 0 \\ -4 & 3 & -1 \\ 5 & -3 & 1 \end{bmatrix} A \quad [R_1 \to R_1 + R_3]$$

$$\Rightarrow \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -4 & 3 & -1 \\ 5 & -3 & 1 \end{bmatrix} A \quad [R_1 \to \tfrac{1}{2}R_1,\ R_3 \to \tfrac{1}{2}R_3]$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & -1 \\ 5/2 & -3/2 & 7/2 \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & -1 \\ 5/2 & -3/2 & 7/2 \end{bmatrix}$$

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ii) $A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$

Let $A = IA$

$$\Rightarrow \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad [R_2 \to 3R_2,\ R_3 \to 3R_3]$$

$\Rightarrow$ ~~3 −2~~ 3

2 7

$$\Rightarrow \begin{bmatrix} 3 & -2 & 3 \\ 6 & 3 & -3 \\ 12 & -9 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} A \quad [R_2 \to R_2 - 2R_1,\ R_3 \to R_3 - 4R_1]$$

$$\Rightarrow \begin{bmatrix} 3 & -2 & 3 \\ 0 & 7 & -9 \\ 0 & -1 & -6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 3 & 0 \\ -4 & 0 & 3 \end{bmatrix} A \quad [R_1 \to 7R_1,\ R_3 \to 7R_3]$$

$$\Rightarrow \begin{bmatrix} 21 & -14 & 21 \\ 0 & 7 & -9 \\ 0 & -7 & -42 \end{bmatrix} = \begin{bmatrix} 7 & 0 & 0 \\ -2 & 3 & 0 \\ -28 & 0 & 21 \end{bmatrix} A \quad [R_1 \to R_1 + 2R_2,\ R_3 \to R_3 + R_2]$$

$$\Rightarrow \begin{bmatrix} 21 & 0 & 3 \\ 0 & 7 & -9 \\ 0 & 0 & -51 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 0 \\ -2 & 3 & 0 \\ -30 & 3 & 21 \end{bmatrix} A \quad [R_1 \to \frac{17}{3} R_1,\ R_2 \to 17R_2,\ R_3 \to -\frac{1}{3} \cdot R_3]$$

$$\Rightarrow \begin{bmatrix} 119 & 0 & 17 \\ 0 & 119 & -153 \\ 0 & 0 & -17 \end{bmatrix} = \begin{bmatrix} 17 & 34 & 0 \\ -34 & 51 & 0 \\ 10 & 1 & 7 \end{bmatrix} A \quad [R_1 \to R_1 + R_3,\ R_2 \to R_2 - 9R_3]$$

$$\Rightarrow \begin{bmatrix} 119 & 0 & 0 \\ 0 & 119 & 0 \\ 0 & 0 & -17 \end{bmatrix} = \begin{bmatrix} 7 & 35 & 7 \\ 56 & 42 & -63 \\ 10 & 1 & 7 \end{bmatrix} A \quad [R_1 \to \frac{1}{119} R_1,\ R_2 \to \frac{1}{119} R_2,\ R_3 \to \frac{-1}{17} R_3]$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \frac{1}{17} \begin{bmatrix} 1 & 5 & 1 \\ 8 & 6 & -9 \\ 10 & -1 & -7 \end{bmatrix} A$$

$$\therefore A^{-1} = \frac{1}{17} \begin{bmatrix} 1 & 5 & 1 \\ 8 & 6 & -9 \\ 10 & -1 & -7 \end{bmatrix}$$

iv)

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

Let $A = IA$

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$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad [R_3 \to R_3 - R_1]$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} A \quad [R_1 \to R_1 - R_2, R_3 \to R_3 - R_2]$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & -3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix} A \quad [R_2 \to 3R_2]$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 6 \\ 0 & 0 & -3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 3 & 0 \\ -1 & -1 & 1 \end{bmatrix} A \quad [R_2 \to R_2 + 2R_3]$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 1 & 2 \\ -1 & -1 & 1 \end{bmatrix} A \quad [R_2 \to \tfrac{1}{3}R_2, R_3 \to \tfrac{-1}{3}R_3]$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2/3 & 1/3 & 2/3 \\ 1/3 & 1/3 & -1/3 \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -2/3 & 1/3 & 2/3 \\ 1/3 & 1/3 & -1/3 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 3 & -3 & 0 \\ -2 & 1 & 2 \\ 1 & 1 & -1 \end{bmatrix}$$

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iv)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 1 & 0 & 2 \end{bmatrix}$$

Let $A = IA$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad [R_2 \to R_2 - 2R_1] \; [R_3 \to R_3 - R_1]$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -2 \\ 0 & -2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} A \quad [R_1 \to 3R_1, R_3 \to 3R_3]$$

$$\Rightarrow \begin{bmatrix} 3 & 6 & 9 \\ 0 & -3 & -2 \\ 0 & -6 & -3 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & 0 & 3 \end{bmatrix} A \quad [R_1 \to R_1 + 2R_2, R_3 \to R_3 - 2R_2]$$

$$\Rightarrow \begin{bmatrix} 3 & 0 & 5 \\ 0 & -3 & -2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 3 \end{bmatrix} A \quad [R_1 \to R_1 - 5R_3, R_2 \to R_2 + 2R_3]$$

$$\Rightarrow \begin{bmatrix} 3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -6 & 12 & -15 \\ 0 & -3 & 6 \\ 1 & -2 & 3 \end{bmatrix} A \quad [R_1 \to \tfrac{1}{3} R_1, R_2 \to -\tfrac{1}{3} R_2]$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 4 & -5 \\ 0 & 1 & -2 \\ 1 & -2 & 3 \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} -2 & 4 & -5 \\ 0 & 1 & -2 \\ 1 & -2 & 3 \end{bmatrix}$$

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Date: 27.08.23

Symmetric and skew-symmetric matrices

i) A square matrices $A = [a_{ij}]_{n \times n}$ is said to be symmetric, if $A' = A$ i.e. $a_{ij} = a_{ji}$ $\forall i$ and j

ii) A square matrix A is said to be skew-symmetric if $A' = A$, i.e. $a_{ij} = a_{ji}$, $\forall i$ and j.

Properties of symmetric and skew-symmetric matrices:

i) Elements of principle diagonals of a skew symmetric matrix are all zero i.e. $a_{ii} = a_{ii} \Rightarrow 2a_{ii} = 0$ or $a_{ii} = 0$, for all values of i.

ii) If A is a square matrix, then

a) $A + A'$ is symmetric.

b) $A - A'$ is skew symmetric matrix.

iii) If A and B are two symmetric (or skew-symmetric) matrices of same order, then $A+B$ is also symmetric (or skew-symmetric)

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v/ If A and B are symmetric matrices of the same order, then the product AB is symmetric, iff $BA = AB$.

vi/ The matrix $B'AB$ is symmetric or skew symmetric according as A is symmetric or skew-symmetric matrix.

x/ If A and B are symmetric matrices of the same order, then

a/ $AB - BA$ is a skew-symmetric and

b/ $AB + BA$ is symmetric.

xi/ For a square matrix A, AA' and $A'A$ are symmetric matrix.

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Elementary operations (Transformation of a matrix)

Any one of the following operations on a matrix is called an elementary transformations.

i/ Interchanging any two rows (or column) denoted by

$$R_i \leftrightarrow R_j \text{ or } C_i \leftrightarrow C_j.$$

ii/ Multiplication of the element of any row (or column) by a non-zero scalar quantity and denoted by

$$R_i \to KR_i \text{ or } C_i \to KC_i$$

iii/ Addition of constant multiple of the elements of any row to the corresponding element of any other row, denoted by

$$R_i \to R_i + KR_j \text{ or } C_i \to C_i + KC_j$$

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Elementary matrix

A matrix obtained from an identity matrix by a single elementary operation is called an elementary matrix.

Equivalent matrix

~~Equi~~ Two matrices A and B are said to be equivalent, if one can be obtained ~~be~~ from the other by a sequence of elementary transformation

The symbols $=$ is used for equivalence.

Example:

$$A = \begin{bmatrix} 1 & -2 & 0 \\ -3 & 2 & 1 \\ 3 & 5 & 3 \end{bmatrix}$$

Then $A' = \begin{bmatrix} 1 & -3 & 3 \\ -2 & 2 & 5 \\ 0 & 1 & 3 \end{bmatrix}$

and $-A' = \begin{bmatrix} -1 & 3 & -3 \\ 2 & -2 & -5 \\ 0 & -1 & -3 \end{bmatrix}$

Thus $A+A' = \begin{bmatrix} 2 & -5 & 3 \\ -5 & 4 & 6 \\ 3 & 6 & 6 \end{bmatrix}$

and $A-A' = \begin{bmatrix} 0 & 1 & -3 \\ -1 & 0 & -4 \\ 3 & 4 & 0 \end{bmatrix}$

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