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Exercise - 7(a)

1 (i) $\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$

Ans - Let $y = \cos^{-1}x$

$\Rightarrow x = \cos y$

$\Rightarrow \frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)} = \frac{1}{-\sin y}$

$\therefore \frac{dy}{dx} = \frac{-1}{\sqrt{1-\cos^2 y}} = \frac{-1}{\sqrt{1-x^2}}$

(5) Let $y = \tan^{-1}x$

$\Rightarrow x = \tan y$

$\Rightarrow \frac{dx}{dy} = \sec^2 y$

$\therefore \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{\sec^2 y} = \frac{1}{1+\tan^2 y} = \frac{1}{1+x^2}$

6) Let $y = \cot^{-1}x$

$\Rightarrow x = \cot y$

$\Rightarrow \frac{dx}{dy} = -\operatorname{cosec}^2 y$

$\therefore \frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)} = \frac{1}{-\operatorname{cosec}^2 y}$

$= \frac{-1}{1+\cot^2 y} = \frac{-1}{1+x^2}$

7)

2. $\sin^{-1} 2x$

Ans: Let $y = \sin^{-1} 2x$

Then $\frac{dy}{dx} = \frac{1}{\sqrt{1-(2x)^2}} \cdot \frac{d}{dx}(2x)$

$\left[\because \frac{d}{dx}(\sin^{-1} u) = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}\right]$

$= \frac{1}{\sqrt{1-4x^2}} \cdot 2 = \frac{2}{\sqrt{1-4x^2}}$

3. $\cot^{-1}\sqrt{x}$

Ans: Let $y = \cot^{-1}\sqrt{x}$

Then $\frac{dy}{dx} = \frac{1}{1+(\sqrt{x})^2} \cdot \frac{d}{dx}(\sqrt{x})$

$\left[\because \frac{d}{dx} \cot^{-1} u = \frac{-1}{1+u^2} \cdot \frac{du}{dx}\right]$

$= \frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}} = -\frac{1}{2\sqrt{x}+x^{3/2}}$

4) $\sec^{-1}(2x+1)$

Ans: Let $y = \sec^{-1}(2x+1)$

$$\frac{dy}{dx} = \frac{1}{(2x+1)\sqrt{(2x+1)^2}} \cdot \frac{d}{dx}(2x+1)$$

$$\left[\because \frac{d}{dx}(\sec^{-1}u) = \frac{1}{|u|\sqrt{u^2-1}} \cdot \frac{du}{dx}\right]$$

$$= \frac{2}{2(2x+1)\sqrt{4x^2+4x}} = \frac{1}{(2x+1)\sqrt{x^2+x}}$$

(5) $\cos^{-1}\sqrt{\frac{1+x}{2}}$

Ans: $y = \cos^{-1}\sqrt{\frac{1+x}{2}}$

$\because$ put $x = \cos\theta$

Then $\frac{1+x}{2} = \frac{1+\cos\theta}{2}$

$= \frac{2\cos^2\frac{\theta}{2}}{2} = \cos^2\frac{\theta}{2}$

$$= \cos^{-1}\cos\frac{\theta}{2} = \frac{\theta}{2} = \frac{1}{2}\cos^{-1}x$$

Then $\frac{dy}{dx} = \frac{-1}{2\sqrt{1-x^2}}$

6/ $\cos^{-1}\left(\dfrac{x-\frac{1}{x}}{x+\frac{1}{x}}\right)$

Ans→ $y=\cos^{-1}\dfrac{x-\frac{1}{x}}{x+\frac{1}{x}}=\cos^{-1}\dfrac{1-\frac{1}{x^2}}{1+\frac{1}{x}}$

[Put $\frac{1}{x}=\tan\theta$

$=\cos^{-1}\dfrac{1-\tan^2\theta}{1+\tan^2\theta}=\cos^{-1}(\cos 2\theta)$

$=2\theta=2\tan^{-1}\dfrac{1}{x}$

Then $\dfrac{dy}{dx}=\dfrac{2x^2}{1+x^2}\cdot\dfrac{d}{dx}\left(\dfrac{1}{x}\right)$

[∵ $\dfrac{d}{dx}(\tan^{-1}u)=\dfrac{1}{1+u^2}\cdot\dfrac{du}{dx}$

$=\dfrac{2x^2}{1+x^2}\cdot\dfrac{-1}{x^2}=\dfrac{-2}{1+x^2}$

7/ $\tan^{-1}(\cos\sqrt{x})$

Ans→ $y=\tan^{-1}(\cos\sqrt{x})$

$\dfrac{dy}{dx}=\dfrac{1}{1+\cos^2\sqrt{x}}\cdot\dfrac{d}{dx}(\cancel{\cos\sqrt{x}})\quad\dfrac{d\cos\sqrt{x}}{dx}\cdot\dfrac{d\sqrt{x}}{dx}$

$=\dfrac{1}{1+\cos^2\sqrt{x}}\cdot(-\sin\sqrt{x})\cdot\dfrac{d}{dx}(\sqrt{x})$

$$\frac{\sin\sqrt{x}}{1+\cos^2\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$$

8/ $x^2 \operatorname{cosec}^{-1}\left(\frac{1}{\ln x}\right)$

Ans→ $y = x^2 \operatorname{cosec}^{-1}\left(\frac{1}{\ln x}\right) = x^2 \cdot \sin^{-1}(\ln x)$

$$\frac{dy}{dx} = \frac{d}{dx}(x^2)\cdot \sin^{-1}(\ln x) + x^2 \cdot \frac{d}{dx}\left[\sin^{-1}(\ln x)\right]$$

$$\left[\sin^{-1}(\ln x)\right]\Big]$$

$$= 2x\sin^{-1}(\ln x) + \frac{x^2}{\sqrt{1-(\ln x)^2}} \cdot \frac{d}{dx}(\ln x)$$

$$= 2\sin^{-1}(\ln x) + \frac{x^2}{\sqrt{1-(\ln x)^2}} \cdot \frac{1}{x}$$

$$= 2\sin^{-1}(\ln x) + \frac{x}{\sqrt{1-(\ln x)^2}}$$

9/ $\cot^{-1}\sqrt{\frac{1-x^2}{x}}$

Ans→ $y = \cot^{-1}\sqrt{\frac{1-x^2}{x}}$ [Put $x = \sin\theta$

$= \cot^{-1} \dfrac{\sqrt{1-\sin^2\theta}}{\sin\theta} = \cot^{-1}\dfrac{\cos\theta}{\sin\theta}$

$= \cot^{-1}\cot\theta = \theta \ \sin^{-1}x$

$\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-x^2}}$

10/ $(x\sin^{-1}x)^{15}$

Ans → $y = (x\sin^{-1}x)^{15}$

$\dfrac{dy}{dx} = 15(x\sin^{-1}x)^{14} \cdot \dfrac{d}{dx}(x\sin^{-1}x)$

$= 15(x\sin^{-1}x)^{14}$

$\left\{\dfrac{d}{dx}(x)\cdot\sin^{-1}x + x\cdot\dfrac{d}{dx}(\sin^{-1}x)\right\}$

$= 15(x\sin^{-1}x)^{14}\left\{\sin^{-1}x + \dfrac{x}{\sqrt{1-x^2}}\right\}$

11/ $\sin^{-1}\sqrt{\dfrac{1-x}{1+x}}$

Ans → $y = \sin^{-1}\sqrt{\dfrac{1-x}{1+x}}$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-\frac{1-x}{1+x}}} \times \frac{d}{dx}\sqrt{\frac{1-x}{1+x}}$$

$$= \frac{\sqrt{1+x}}{\sqrt{2x}} \cdot \frac{1}{\sqrt{\frac{1-x}{1+x}}} \cdot \frac{d}{dx}\left(\frac{1-x}{1+x}\right)$$

$$= \frac{\sqrt{1+x}}{\sqrt{2x}} \cdot \frac{\sqrt{1+x}}{2\sqrt{1-x}} \cdot \frac{-1(1+x)-(1-x)}{(1+x)^2}$$

$$= \frac{1+x}{2\sqrt{2}\sqrt{x-x^2}} \cdot \frac{-2}{(1+x)^2}$$

$$= -\frac{1}{\sqrt{2}\sqrt{x-x^2}\cdot(1+x)}$$