$$= \frac{1-\frac{1}{e^{\infty}}}{1+\frac{1}{e^{\infty}}} = \frac{1}{1} = 1$$

As L.H.L $\neq$ R.H.L

So, the fcn is discontinueus at $x=0$.

Exercise 7(b)

i) e^{3x}

Let $y = e^{3x}$

Then $$\frac{dy}{dx} = \lim_{\delta x\to 0} \frac{e^{3(x+\delta x)} - e^{3x}}{\delta x}$$

$$= \lim_{\delta x\to 0} \frac{e^{3x}(e^{3\delta x}-1)}{\delta x}$$

$$= \lim_{\delta x\to 0} e^{3x} \cdot 3\frac{e^{3\delta x}-1}{3\delta x}$$

$$= 3e^{3x} \cdot \lim_{\delta x\to 0} \frac{e^{3\delta x}-1}{3\delta x} = 3e^{3\cdot x}$$

$$\left[\because \lim_{\theta\to 0} \frac{e^{\theta}-1}{\theta} = 1\right]$$

(ii) $2x^2$

Let $y = 2x^2$

Then $$\frac{dy}{dx} = \lim_{\delta x\to 0} \frac{2(x+\delta x)^2 - 2x^2}{\delta x}$$

$$= \lim_{\delta x\to 0} \frac{2x^2 + 2x\delta x + \delta x^2 - 2x^2}{\delta x}$$

$$= \lim_{\delta x \to 0} 2^{x^2} \left[\frac{2^{(2x+\delta x)\delta x} - 1}{\delta x} \right]$$

$$= 2^{x^2} \lim_{\delta x \to 0} \left\{ (2x+\delta x) \frac{2^{(2x+\delta x)\delta x} - 1}{(2x+\delta x)\cdot \delta x} \right\}$$

$$= 2^{x^2} \lim_{\delta x \to 0} (2x+\delta x) \cdot \lim_{\delta x \to 0} \frac{2^{(2x+\delta x)\delta x} - 1}{(2x+\delta x)\delta x}$$

$$= 2^{x^2} \cdot 2x \ln 2 \left[\because \lim_{y \to 0} \frac{e^y - 1}{y} = 1 \right.$$

$$= 2^{x^2+1} x \ln 2 \left[\lim_{y \to 0} \frac{a^y - 1}{y} = \ln a \right.$$

(iii) $\ln(3x+1)$

Ans: Let $y = \ln(3x+1)$

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \left\{ \frac{\ln(3x+3\delta x+1) - \ln(3x+1)}{\delta x} \right\}$$

$$= \lim_{\delta x \to 0} \frac{1}{\delta x} \ln \frac{3x+3\delta x+1}{3x+1}$$

$$= \lim_{\delta x \to 0} \frac{1}{\delta x} \cdot \ln \left(1 + \frac{3\delta x}{3x+1}\right)$$

$$= \lim_{\delta x \to 0} \left\{ \frac{3}{3x+1} \cdot \frac{3x+1}{3\delta x} \cdot \ln \left(1 + \frac{3\delta x}{3x+1}\right) \right\}$$

$$= \frac{3}{3x+1} \times \lim_{\delta x \to 0} \ln \left(1 + \frac{3\delta x}{3x+1}\right)^{\frac{3x+1}{3\delta x}}$$

$$\left[\begin{array}{l} \text{put } t = \frac{3\delta x}{3x+1} \\ \text{As } \delta x \to 0, t \to 0 \end{array} \right.$$

$$= \frac{3}{3x+1} \lim_{t \to 0} \ln (1+t)^{\frac{1}{t}}$$

$$= \frac{3}{3x+1} \ln e \left[\because \lim_{t \to 0} \ln (1+t)^{\frac{1}{t}} = e \quad \boxed{\ln e = 1} \right.$$

$$= \frac{3}{3x+1}$$

iv) $\log_x 5$ $\left(\text{Hint: } \log_x 5 = \frac{\ln 5}{\ln 5} \right)$

Ans→ Let $y = \log_x 5 = \frac{\ln 5}{\ln x}$

$$\text{Then } \frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\frac{\ln 5}{\ln (x+\delta x)} - \frac{\ln 5}{\ln x}}{\delta x}$$

$$= \ln 5 \cdot \lim_{\delta x \to 0} \frac{\ln x - \ln (x+\delta x)}{\delta x \cdot \ln x \cdot \ln (x+\delta x)}$$

$$= -\ln 5 \cdot \lim_{\delta x \to 0} \frac{\ln \left(1 + \frac{\delta x}{x}\right)}{\delta x \cdot \ln x \cdot \ln (x+\delta x)}$$

$$= -\ln 5 \cdot \lim_{\delta x \to 0} \frac{1}{\ln x \cdot \ln (x+\delta x)}$$

$$\times \cancel{\lim_{\delta x \to 0}} \cancel{\delta x} \cancel{\ln}$$

$$\times \lim_{\delta x \to 0} \frac{1}{\delta x} \ln \left(1 + \frac{\delta x}{x}\right)$$

$$= -\frac{\ln 5}{(\ln x)^2} \times \frac{1}{x} \times \lim_{\delta x \to 0} \ln \left(1 + \frac{\delta x}{x}\right)^{\frac{x}{\delta x}}$$

$$= -\frac{\ln 5}{x (\ln x)^2} \left[\because \lim_{t \to 0} (1+t)^{\frac{1}{t}} = e \text{ and } \ln e = 1 \right.$$

e ର power

$e^{\ln x} = x$

Ans ⇒ Let $y = \ln \sin x$

Then $e^{y} = \sin x \quad \ldots (1)$

Let δx be an increment of x and δy be the corresponding increment of y.

Then $e^{y+\delta x} = \sin(x+\delta x) \text{ --- } (2)$

Substracting (1) from (2) we get

$e^{y+\delta y} - e^{y} = \sin(x+\delta y) - \sin x$

$\Rightarrow \dfrac{e^{y+\delta y} - e^{y}}{\delta x} = \dfrac{\sin(x+\delta x) - \sin x}{\delta x}$

$\Rightarrow \lim\limits_{\delta x \to 0} \left[e^{y} \left(\dfrac{e^{\delta y} - 1}{\delta y} \right) \dfrac{\delta y}{\delta x} \right]$

$= \lim\limits_{\delta x \to 0} \dfrac{\sin(x+\delta x) - \sin x}{\delta x}$

$\Rightarrow e^{y} \dfrac{dy}{dx} = \lim\limits_{\delta x \to 0} \dfrac{2 \sin \frac{\delta x}{2} \cdot \cos\left(x + \frac{\delta x}{2}\right)}{\delta x}$

$\left[\because \lim\limits_{\delta y \to 0} \dfrac{e^{\delta y} - 1}{\delta y} = 1 \text{ and } \lim\limits_{\delta x \to 0} \dfrac{\delta y}{\delta x} = \dfrac{dy}{dx} \right.$

We note that when $\delta x \to 0$, $\delta y \to 0$

$\Rightarrow e^{y}\frac{dy}{dx} = \lim_{\delta x \to 0}\left[\frac{\sin\frac{\delta x}{2}}{\frac{\delta x}{2}} \times \cos\left(x+\frac{\delta x}{2}\right)\right] = \cos x$

$\Rightarrow \sin x \cdot \frac{dy}{dx} = \cos x$

$\Rightarrow \frac{dy}{dx} = \frac{\cos x}{\sin x} = \cot x$

vi) Let $y = x^2 a^{2x}$

Then $y + \delta y = (x+\delta x)^2 \cdot a^{2(x+\delta x)}$

$\Rightarrow \delta y = (x+\delta x)^2\, a^{2(x+\delta y)} - x^2 a^{2x}$

$\Rightarrow \frac{\delta y}{\delta x} = \frac{(x^2 + 2x\delta x + \delta x^2)\,a^{2x+\delta x} - x^2 a^{2x}}{\delta x}$

$\Rightarrow \frac{dy}{dx} = \lim_{\delta x \to 0} \frac{a^{2x}\{(x^2 + 2x\delta x + \delta x^2)\,a^{2\delta x} - x^2\}}{\delta x}$

$\Rightarrow a^{2x} \lim_{\delta x \to 0} \frac{x^2(a^{2\delta x} - 1) + 2x\cdot\delta x\cdot a^{2\delta x} + \delta x^2 \cdot a^{2\delta x}}{\delta x}$

$\Rightarrow a^{2x}\left\{\lim_{\delta x \to 0} \frac{x^2(a^{2\delta x} - 1)}{\delta x} + \lim_{\delta x \to 0} \frac{2x\cdot\delta x\cdot a^{2\delta x} + \delta x^2 \cdot a^{2\delta x}}{\delta x}\right\}$

$= a^{2x}\left\{2x^2 \lim_{\delta x \to 0} \frac{a^{2\delta x} - 1}{2\delta x} + \lim_{\delta x \to 0} (2x\cdot a^{2\delta x} + \delta x \cdot a^{2\delta x})\right\}$

$= a^{2x}(2x^2 \cdot \ln a + 2x) = 2x \cdot a^{2x}(x \ln a + 1)$