Exercise - 7(e)

1. $\sin^{-1} 2x\sqrt{1-x^2}$

Ans → $y = \sin^{-1} 2x\sqrt{1-x^2}$ put $x = \sin\theta$

$= \sin^{-1}(2\sin\theta \cdot \cos\theta)$

$= \sin^{-1} \sin 2\theta = 2\theta = 2\sin^{-1} x$

$\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}$

2. $\tan^{-1} \frac{2x}{1-x^2}$

Ans → $y = \tan^{-1} \frac{2x}{1-x^2}$ [put $x = \tan\theta$

$= \tan^{-1} \dfrac{2\tan\theta}{1-\tan^2\theta} = \tan^{-1}(\tan 2\theta)$

$= 2\theta = 2\tan^{-1}x$

$\therefore \dfrac{dy}{dx} = \dfrac{2}{1+x^2}$

3/ $\tan^{-1}\sqrt{\dfrac{1-t}{1+t}}$

Ans :- $y = \tan^{-1}\sqrt{\dfrac{1-t}{1+t}}$ [put $t = \cos\theta$

$= \tan^{-1}\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}$

$= \tan^{-1}\sqrt{\dfrac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}}$

$= \tan^{-1}\left(\tan\dfrac{\theta}{2}\right) = \dfrac{\theta}{2} = \dfrac{1}{2}\cos^{-1}x$

$\therefore \dfrac{dy}{dx} = \dfrac{-1}{2\sqrt{1-t^2}}$

4/ $\left[\left(\dfrac{1+t^2}{1-t^2}\right)^2 - 1\right]^{\frac{1}{2}}$

Ans :- $y = \left[\left(\dfrac{1+t^2}{1-t^2}\right)^2 - 1\right]^{\frac{1}{2}}$

$$= \left[\frac{(1+t^2)^2 - (1-t^2)^2}{(1-t^2)^2}\right]^{\frac{1}{2}}$$

$$= \left[\frac{4t^2}{(1-t^2)^2}\right]^{\frac{1}{2}} = \frac{2t}{1-t^2}$$

$$\therefore \frac{dy}{dt} = \frac{(1-t^2)\cdot 2 - 2t\,(-2t)}{(1-t^2)^2}$$

$$= \frac{2+2t^2}{(1-t^2)^2} = \frac{2(1+t^2)}{(1-t^2)^2}$$

5. $\tan^{-1}\left(\frac{\sqrt{x}+\sqrt{a}}{1-\sqrt{xa}}\right)$

Ans $y = \tan^{-1}\frac{\sqrt{x}+\sqrt{a}}{1-\sqrt{xa}} = \tan^{-1}\sqrt{x} + \tan^{-1}\sqrt{a}$

$$\frac{dy}{dx} = \frac{1}{1+x}\cdot\frac{d}{dx}(\sqrt{x})$$

$\because \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}\cdot\frac{du}{dx}$

and ~~as~~ is a constant $\tan^{-1}a$ is constant and so $\frac{d}{dx}(\tan^{-1}a) = 0$

$= \frac{1}{2\sqrt{x}(1+x)}$

6/ $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Ans → $y = \sin^{-1}\frac{2x}{1+x^2}$ [put $x = \tan\theta$

$= \sin^{-1}\frac{2\tan\theta}{1+\tan^2\theta} = \sin^{-1}\sin 2\theta$

$= 2\theta = 2\tan^{-1}x$

$\therefore \frac{dy}{dx} = \frac{2}{1+x^2}$

7/ $\sec^{-1}\left(\frac{\sqrt{a^2+x^2}}{a}\right)$

Ans → $y = \sec^{-1}\frac{\sqrt{a^2+x^2}}{a}$ [put $x = a\tan\theta$

$= \sec^{-1}\frac{\sqrt{a^2+a^2\tan^2\theta}}{a}$

$= \sec^{-1}\sec\theta = \theta = \tan^{-1}\frac{x}{a}$

$\frac{dy}{dx} = \frac{1}{1+\left(\frac{x}{a}\right)^2}\cdot\frac{d}{dx}\left(\frac{x}{a}\right)$

$= \frac{a^2}{a^2+x^2}\cdot\frac{1}{a} = \frac{a}{a^2+x^2}$

8. $\sin^{-1} \frac{2\sqrt{t^2-1}}{t^2}$

Ans $\Rightarrow$ $y = \sin^{-1} \frac{2\sqrt{t^2-1}}{t^2}$ [put $t = \sec\theta$

$= \sin^{-1}\left(\frac{2\sqrt{\sec^2\theta - 1}}{\sec^2\theta}\right) = \sin^{-1}\left(\frac{2\tan\theta}{\sec^2\theta}\right)$

$= \sin^{-1}(2\sin\theta\cos\theta)$

$= \sin^{-1}\sin 2\theta = 2\theta = 2\sec^{-1} t$

$\therefore \frac{dy}{dx} = \frac{2}{|t|\sqrt{(t^2-1)}}$

9. $\cos^{-1}\left(\frac{1-t^2}{1+t^2}\right)$

Ans $\Rightarrow$ $y = \cos^{-1}\left(\frac{1-t^2}{1+t^2}\right)$ (put $t = \tan\theta$)

$= \cos^{-1} \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$= \cos^{-1}\cos 2\theta = 2\theta = 2\tan^{-1} t$

$\therefore \frac{dy}{dx} = \frac{2}{1+t^2}$

10. $\cos^{-1}(2t^2 - 1)$

Ans $\Rightarrow$ $y = \cos^{-1}(2t^2-1)$ [put $t = \cos\theta$

$$= \cos^{-1}(2\cos^2\theta - 1)$$

$$= \cos^{-1}\cos 2\theta = 2\theta = 2\cos^{-1}t$$

$$\therefore \frac{dy}{dx} = -\frac{2}{\sqrt{1-t^2}}$$