$$\therefore \frac{dy}{dx} = -\frac{2}{\sqrt{1-t^2}}$$

Exercise - 7(f) Date :- 29.09.23

1. x^x

Ans :- Let $y = x^x$

$$\Rightarrow \frac{d}{dx}(\ln y) = \frac{d}{dx}(x \cdot \ln x)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \ln + x \cdot \frac{1}{x} \quad \ln x + 1 = 1 + \ln x$$

$$\Rightarrow \frac{dy}{dx} = y(1+\ln x) = x^x(1+\ln x) = x^x \ln(ex)$$

(2) $\left(1+\frac{1}{x}\right)$

Ans :- Let $y = \left(1+\frac{1}{x}\right)^x$

Then $\ln y = x \cdot \ln\left(1+\frac{1}{x}\right)$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \ln\left(1+\frac{1}{x}\right) + x \cdot \frac{1}{1+\frac{1}{x}} \cdot \frac{d}{dx}\left(1+\right.$$

$$\Rightarrow = \ln\left(\frac{x+1}{x}\right) + \frac{x^2}{1+x} \cdot \left(-\frac{1}{x^2}\right)$$

$$\Rightarrow = \ln\left(\frac{1+x}{x}\right) - \frac{1}{1+x}$$

$\therefore \frac{dy}{dx} = y\left\{ \ln\frac{1+x}{x} - \frac{1}{1+x} \right\}$

$= \left(1+\frac{1}{x}\right)^x \left\{ \ln\frac{1+x}{x} - \frac{1}{1+x} \right\}$

3/ $x^{\sin x}$

Ans ⇒ Let $y = x^{\sin x}$

$\Rightarrow \ln y = \sin x \cdot \ln x$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \cos x \ln x + \sin x \cdot \frac{1}{x}$

$\Rightarrow \frac{dy}{dx} = x^{\sin x}\left(\cos x \cdot \ln x + \frac{\sin x}{x}\right)$

4/ $(\log x)^{\tan x}$

Ans ⇒ $y = (\log x)^{\tan x}$

$\Rightarrow \log y = \tan x \cdot \log(\log x)$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \sec^2 x \log(\log x) + \tan x \cdot \frac{1}{\log x} \cdot \frac{1}{x}$

$\Rightarrow \frac{dy}{dx} = (\log x)^{\tan x}\left\{ \sec^2 x \log \log x + \frac{\tan x}{x \log x} \right\}$

5/ $2^{(2^x)}$

Ans ⇒ $y = 2^{(2^x)}$

$\Rightarrow \ln y = 2^x \cdot \ln 2$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = 2^x \cdot \ln 2 \cdot \ln 2$

$\Rightarrow \frac{dy}{dx} = 2^{(2^x)} \cdot 2^x \cdot (\ln 2)^2$

6/ $(1+\sqrt{x})^{x^2}$

Ans ⇒ $y = (1+\sqrt{x})^{x^2}$

$\Rightarrow \ln y = x^2 \ln(1+\sqrt{x})$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = 2x \ln(1+\sqrt{x}) + x^2 \cdot$

$\frac{1}{1+\sqrt{x}} \cdot \frac{d}{dx}(1+\sqrt{x})$

$\frac{d\sqrt{x}}{dx}$

$\frac{1}{2\sqrt{x}}$

$\boxed{\ln e = 1}$

$\left[\because \frac{d}{dx}(\ln u) = \frac{1}{u}\cdot\frac{du}{dx}\right.$ by chain rule

$= 2x \cdot \ln(1+\sqrt{x}) + \frac{x^2}{1+\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$

$\frac{dy}{dx} = (1+\sqrt{x})^{x^2}$

$\left\{2x\ln(1+\sqrt{x}) + \frac{x^{3/2}}{2(1+\sqrt{x})}\right\}$

7/ $(\sin^{-1}x)^{\sqrt{1-x^2}}$

Ans:- $y = (\sin^{-1}x)^{\sqrt{1-x^2}}$

$\Rightarrow \ln y = \sqrt{1-x^2} \cdot \ln(\sin^{-1}x)$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}(\sqrt{1-x^2}) \cdot \ln(\sin^{-1}x)$

$+ \sqrt{1-x^2} \cdot \frac{d}{dx}(\ln \sin^{-1}x)$

$= \frac{1}{2\sqrt{1-x^2}} \cdot \frac{d}{dx}(1-x^2) \cdot \ln(\sin^{-1}x)$

$+ \sqrt{1-x^2} \cdot \frac{1}{\sin^{-1}x} \frac{d}{dx}(\sin^{-1}x)$

$= \frac{-2x}{2\sqrt{1-x^2}} \cdot \ln \sin^{-1}x + \frac{\sqrt{1-x^2}}{\sin^{-1}x} \cdot \frac{1}{\sqrt{1-x^2}}$

$\therefore \frac{dy}{dx} = (\sin^{-1}x)^{\sqrt{1-x^2}}$

$\left\{ \frac{-x \ln(\sin^{-1}x)}{\sqrt{1-x^2}} + \frac{1}{\sin^{-1}x} \right.$

8/ $(\tan x)^{\log x^3}$

Ans:- $y = (\tan x)^{\log x^3}$

$\Rightarrow \log y = \log x^3 \cdot \log(\tan x)$

$= 3 \log x \cdot \log(\tan x)$

$\therefore \frac{1}{y}\frac{dy}{dx}$

$= 3\left\{\frac{1}{x} \cdot \log \tan x + \log x \cdot \frac{1}{\tan x} \cdot \sec^2 x\right\}$

$= 3\left\{\frac{\log \tan x}{x} + \frac{\sec^2 x \cdot \log x}{\tan x}\right\}$

$\therefore \frac{dy}{dx} = (\tan x)^{\log 3}\left\{\frac{\log \tan x}{x} + \frac{\sec^2 x}{\tan x}\right\}$

9. $x^{1/x} + (\sin x)^x$

Ans $\Rightarrow y = x^{\frac{1}{x}} + (\sin x)^x$

Put $u = x^{\frac{1}{x}}$, $v = (\sin x)^x$

Then $\ln u = \frac{\ln x}{x}$

$\Rightarrow \frac{1}{u}\frac{du}{dx} = \frac{1}{x} \cdot \frac{x - \ln x \cdot 1}{x^2}$

$\Rightarrow \frac{du}{dx} = u\frac{1 - \ln x}{x^2} = x^{\frac{1}{x}} \cdot \frac{1 - \ln x}{x^2}$

$\Rightarrow$ Again $\ln y = x \cdot \ln \sin x$

$\Rightarrow \frac{1}{v}\frac{dv}{dx} = \ln \sin x + x \cdot \frac{1}{\sin x} \cdot \cos x$

$\Rightarrow \frac{dv}{dx} = v(\ln\sin x + x\cot x)$

$= (\sin x)^x \cdot (\ln\sin x + x\cdot\cot x)$

From (1) we get $y = u + v$

$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

$= x^{\frac{1}{x}} \frac{1-\ln x}{x^2} + (\sin x)^x(\ln\sin x + x\cot x)$

10. $(\cos x)^x + x^{\cos x}$

Ans $\Rightarrow y = (\cos x)^x + x^{\cos x}$

Then $\ln u = x \cdot \ln\cos x$

$\Rightarrow \frac{d\ln u}{du}\cdot\frac{du}{dx} = \frac{dx}{dx}\ln\cos x + \frac{d\ln\cos x}{d\cos x}\cdot\frac{d\cos x}{dx}\cdot x$

$\Rightarrow \frac{1}{u}\frac{du}{dx} = \ln\cos x + x\frac{1}{\cos x}(-\sin x)$

$\Rightarrow \frac{du}{dx} = u(\ln\cos x - x\tan x)$

$= (\cos x)^x(\ln\cos x - x\tan x)$

Again $\ln v = \cos x \cdot \ln x$

$\Rightarrow \frac{1}{v}\frac{dv}{dx} = -\sin x\cdot\ln x + \frac{\cos x}{x}$

$\Rightarrow \frac{dv}{dx} = x^{\cos x}\cdot\left\{\frac{\cos x}{x} - \sin x\cdot\ln x\right\}$

Then $y = u + v$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$= (\cos x)^x \cdot (\ln \cos x - x\tan x)$$

$$+ x^{\cos x} \cdot \left(\frac{\cos x}{x} - \sin x \cdot \ln x\right)$$

11) $(x^2+1)^{2/3}(3x+1)^{1/4}\sqrt{x}$

Ans $\Rightarrow$ $y = (x^2+1)^2 \cdot (3x+1)^{\frac{1}{4}} \cdot \sqrt{x}$

since: $\ln abc = \ln a + \ln b$

$\ln y = \ln\left((x^2+1)^{\frac{2}{3}} \cdot (3x+1)^{\frac{1}{4}} \cdot \sqrt{x}\right)$

$\Rightarrow \ln(x^2+1)^{\frac{2}{3}} + \ln(3x+1)^{\frac{1}{4}} + \ln\sqrt{x}$

$$\Rightarrow \ln y = \frac{2}{3}\ln(x^2+1) + \frac{1}{4}\ln(3x+1) + \frac{1}{2}\ln x$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{2}{3} \cdot \frac{2x}{x^2+1} + \frac{3}{4} \cdot \frac{1}{3x+1} + \frac{1}{2x}$$

$$\therefore \frac{dy}{dx} = (x^2+1)^{\frac{2}{3}}(3x+1)^{\frac{1}{4}}\sqrt{x}$$

$$\left\{\frac{4x}{3(x^2+1)} + \frac{3}{4(3x+1)} + \frac{1}{2x}\right\}$$

12) $\dfrac{(x+1)(x+2)^2(x+3)^3}{(x-1)(x-2)^2(x-3)^3}$

Ans $\Rightarrow$ $y = \dfrac{(x+1)(x+2)^2(x+3)^3}{(x-1)(x-2)^2(x-3)^3}$

$$\Rightarrow \ln y = \ln(x+1) + 2\ln(x+2) + 3\ln(x+3)$$

$$- \ln(x-1) - 2\ln(x-2) - 3\ln(x-3)$$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{x+1} + \frac{2}{x+2} + \frac{3}{x+3}$

$- \frac{1}{x-1} \frac{2}{x-2} \frac{3}{x-3}$

$\therefore \frac{dy}{dx} = \frac{(x+1)(x+2)^2(x+3)^3}{(x+1)(x-2)^2(x-3)^3}$

$\times \left\{\frac{1}{x+1} + \frac{2}{x+2} + \frac{3}{x+3} - \frac{1}{x-1} - \frac{2}{x-2} \frac{3}{x-3}\right\}$

13) $(\sin x)^x \sqrt{\sin x}\,(1+x^2)^{\frac{1}{2}+x}$

Ans $\Rightarrow (\sin x)^x \sqrt{\sin x}\,(1+x^2)^{\frac{1}{2}+x}$

$\Rightarrow \ln y = x \cdot \ln \sin x + \frac{1}{2} \ln \sin x$

$+ \left(\frac{1}{2}+x\right) \ln (1+x^2)$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \ln \sin x + \frac{x}{\sin x} \cdot \cos x + \frac{1}{2} \cdot \frac{\cos x}{\sin x}$

$+ \ln (1+x^2) + \left(\frac{1}{2}+x\right) \frac{1}{1+x^2} \cdot 2x$

$\Rightarrow \frac{dy}{dx} = dy \{\ln \sin x + x \cdot \cot x$

$+ \frac{1}{2} \cos x + \ln (1+x^2) + \frac{(1+2x) x}{1+x^2}\}$

$= (\sin x)^x \cdot \sqrt{\sin x}\,(1+x^2)^{\frac{1}{2}+x}$

$\{\ln \sin x + (x+\frac{1}{2}) \cot x$

$+ \ln(1+x^2) + \frac{(1+2x)x}{1+x^2}\}$

14/ $(\sec x + \tan x)^{\cot x}$

Ans $\Rightarrow y = (\sec x + \tan x)^{\cot x}$

$\Rightarrow \ln y = \cot x \cdot \ln(\sec x + \tan x)$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = -\operatorname{cosec}^2 x \cdot \ln(\sec x + \tan x)$

$+ \cot x \cdot \frac{1}{\sec x + \tan x} \times (\sec x \cdot \tan x + \sec$

$= -\operatorname{cosec}^2 x \ln(\sec x + \tan x)$

$+ \cot x \cdot \sec x$

$\frac{dy}{dx}$

$= y\{-\operatorname{cosec}^2 x \cdot \ln(\sec x + \tan x) + \operatorname{cosec}$

$= (\sec x + \tan x)^{\cot x}$

$\{\operatorname{cosec} x - \operatorname{cosec}^2 x \cdot \ln(\sec x + \tan$

15/ $(2^{\sqrt{x}})^{1+\sqrt{x}}$

Ans $\Rightarrow y = (2^{\sqrt{x}})^{1+\sqrt{x}} = 2^{\sqrt{x}+x}$

$\Rightarrow \ln y = (\sqrt{x}+x) \ln 2$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \ln 2 \cdot \left(1+\frac{1}{2\sqrt{x}}\right)$$

$$\Rightarrow \frac{dy}{dx} = 2^{\sqrt{x}+x} \cdot \ln 2\left(1+\frac{1}{2\sqrt{x}}\right)$$

<u>method 2</u>

$$y = \left(2^{\sqrt{x}}\right)^{1+\sqrt{x}} = 2^{\sqrt{x}+x}$$

$$\Rightarrow \frac{dy}{dx} = 2^{\sqrt{x}+x}(\ln 2)\frac{d}{dx}(\sqrt{x}+x)$$

$$= 2^{\sqrt{x}+x}(\ln 2)\left(\frac{1}{2\sqrt{x}}+1\right)$$