Exercise - 7(g) Date → 30.09.23

1. $\frac{dy}{dx}$

$xy^2 + x^2y + 1 = 0$

Ans → $xy^2 + x^2y + 1 = 0$

$\Rightarrow \frac{d}{dx} xy^2 + \frac{d}{dx}(x^2y) + \frac{d}{dx}(1) = 0$

$\Rightarrow \frac{dx}{dx} \cdot y^2 + x \cdot \frac{d}{dx}(y^2) + \frac{d}{dx}(x)^2 \cdot y + x^2 \cdot \frac{dy}{dx} = 0$

$\Rightarrow y^2 + 2x \cdot y \frac{dy}{dx} + 2xy + x^2 \frac{dy}{dx} = 0$

$\Rightarrow (2xy + x^2)\frac{dy}{dx} = -(y^2 + 2xy)$

$\Rightarrow \dfrac{dy}{dx} = -\dfrac{y^2+2xy}{x^2+2xy}$

(2) $x^{\frac{1}{2}} y^{-\frac{1}{2}} + x^{\frac{3}{2}} y^{-\frac{3}{2}} = 0$

Ans $\Rightarrow x^{\frac{1}{2}} y^{-\frac{1}{2}} + x^{\frac{3}{2}} y^{-\frac{3}{2}} = 0$

$\Rightarrow \left(\dfrac{x}{y}\right)^{\frac{1}{2}} = -\left(\dfrac{x}{y}\right)^{\frac{3}{2}}$

$\Rightarrow \dfrac{x}{y} = \left(\dfrac{x}{y}\right)^3$

$\Rightarrow \dfrac{x}{y} = \pm 1$ or $\dfrac{x}{y} = 0$

$\Rightarrow \dfrac{y - x\frac{dy}{dx}}{y^2} = 0$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{y}{x}$

3 $\Rightarrow x^2 + 3y^2 = 5$

Ans $\Rightarrow x^2 + 3y^2 = 5$

$\Rightarrow \dfrac{d}{dx}(x^2) + 3\dfrac{d}{dx}(y^2) = 0$

$\Rightarrow 2x + 6y\dfrac{dy}{dx} = 0 \Rightarrow \dfrac{dy}{dx} = -\dfrac{x}{3y}$

4/ $y^2 \cot x = x^2 \cot y$.

Ans/ $y^2 \cot x = x^2 \cot y$

$\Rightarrow \frac{d}{dx}(y^2 \cot x) = \frac{d}{dx}(x^2 \cot y)$

$\Rightarrow 2y \frac{dy}{dx} \cdot \cot x + y^2 (-\operatorname{cosec}^2 x)$

$= 2x \cot y + x^2 \cdot (-\operatorname{cosec}^2 y) \cdot \frac{dy}{dx}$

$\Rightarrow (2y \cot x + x^2 \operatorname{cosec}^2 y) \cdot \frac{dy}{dx}$

$= y^2 \operatorname{cosec}^2 x + 2x \cot y$

$\Rightarrow \frac{dy}{dx} = \frac{y^2 \operatorname{cosec}^2 x + 2x \cot y}{2y \cot x + x^2 \operatorname{cosec}^2 y}$

5/ $y = \tan xy$

Ans/ $y = \tan xy$

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx} \tan(xy) = \sec^2(xy) \cdot \frac{d}{dx}(xy)$

$= \sec^2(xy) \cdot (y + \frac{dy}{dx})$

$= y \sec^2(xy) + x \sec^2 xy \cdot \frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} \{1 - x \sec^2(xy)\} = y \sec^2(xy)$

$$\Rightarrow \frac{dy}{dx} = \frac{y\sec^2(xy)}{1 - x\sec^2(xy)}$$

6. $x = y\ln(xy)$.

Ans $\Rightarrow$ $x = y\ln(xy)$

$$\Rightarrow \frac{d}{dx}(x) = \frac{d}{dx}\{y\cdot\ln(xy)\}$$

$$\Rightarrow 1 = \frac{dy}{dx}\cdot\ln(xy) + y\,\frac{1}{xy}\cdot\frac{d}{dx}(xy)$$

$$= \frac{dy}{dx}\cdot\ln(xy) + \frac{1}{x}\cdot\left(y + x\frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx}\{\ln(xy) + 1\} = 1 - \frac{y}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1 - \frac{y}{x}}{1 + \ln(xy)} = \frac{x - y}{x\ln(exy)}$$

7. $e^{xy} + y\sin x = 1$

Ans $\Rightarrow$ $e^{xy} + y\sin x = 1$

$$\Rightarrow \frac{d}{dx}(e^{xy}) + \frac{d}{dx}(y\sin x) = 0$$

$$\Rightarrow e^{xy}\cdot\frac{d}{dx}(xy) + \frac{dy}{dx}\cdot\sin x + y\frac{d}{dx}(\sin x)$$

$$\Rightarrow e^{xy}\left(y + x\frac{dy}{dx}\right) + \sin x\frac{dy}{dx} + y\cos x = 0$$

$$\Rightarrow (xe^{xy} + \sin x)\frac{dy}{dx} = -(ye^{xy} + y\cos x)$$

$$\frac{dy}{dx} = \frac{ye^{xy} + y\cos x}{xe^{xy} + \sin x}$$

5/ $\ln\sqrt{x^2+y^2} = \tan^{-1}\frac{y}{x}$

Ans $\ln\sqrt{x^2+y^2} = \tan^{-1}\frac{y}{x}$

$$\Rightarrow \frac{d}{dx}\left(\ln\sqrt{x^2+y^2}\right) = \frac{d}{dx}\left(\tan^{-1}\frac{y}{x}\right)$$

$$\Rightarrow \frac{1}{\sqrt{x^2+y^2}} \cdot \frac{d}{dx}\sqrt{x^2+y^2} = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{d}{dx}\left(\frac{y}{x}\right)$$

$$\Rightarrow \frac{1}{\sqrt{x^2+y^2}} \cdot \frac{1}{2\sqrt{x^2+y^2}} \frac{d}{dx}(x^2+y^2)$$

$$= \frac{x^2}{x^2+y^2}\left\{\frac{\frac{dy}{dx}\cdot x - y}{x^2}\right\}$$

$$\Rightarrow \frac{1}{2(x^2+y^2)}\left(2x + 2y\frac{dy}{dx}\right)$$

$$= \frac{x}{x^2+y^2}\frac{dy}{dx} - \frac{y}{x^2+y^2}$$

$$\Rightarrow \frac{y-x}{x^2+y^2}\frac{dy}{dx} = -\frac{x+y}{x^2+y^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x+y}{x-y}$$

9/ $y^x = x^{\sin y}$

Ans→ $y^x = x^{\sin y}$

$\Rightarrow x \cdot \ln y = \sin y \cdot \ln x$

$\Rightarrow \frac{d}{dx}(x \cdot \ln y) = \frac{d}{dx}(\sin y \cdot \ln x)$

$\Rightarrow \frac{d}{dx}(x) \cdot \ln y + x \cdot \frac{d}{dx}(\ln y)$

$= \frac{d}{dx}(\sin y) \cdot \ln x + \sin y \cdot \frac{d}{dx}(\ln x)$

$\Rightarrow \ln y + \frac{x}{y}\frac{dy}{dx} = \cos y \cdot \ln x \frac{dy}{dx} + \sin y \frac{1}{x}$

$\Rightarrow \left(\frac{x}{y} - \cos y \ln x\right)\frac{dy}{dx} = \frac{\sin y}{x} - \ln y$

$\Rightarrow \frac{dy}{dx} = \frac{\frac{\sin y}{x} - \ln y}{\frac{x}{y} - \cos y \cdot \ln x} = \frac{y}{x}\frac{(\sin y - x \ln y)}{(x - y \cos y \ln x)}$

10/ If $\sin(x+y) = y\cos(x+y)$ then prove that

$\frac{dy}{dx} = -\frac{1+y^2}{y^2}$.

Ans→ given $\sin(x+y) = y\cos(x+y)$

$\Rightarrow y = \tan(x+y)$

$\Rightarrow \frac{dy}{dx} = \sec^2(x+y)\frac{d}{dx}(x+y)$

$\Rightarrow \frac{dy}{dx} = \{1+\tan^2(x+y)\}\left(1+\frac{dy}{dx}\right)$

$\Rightarrow \frac{dy}{dx} = (1+y^2)\left(1+\frac{dy}{dx}\right)$ [by eqn (1)

$\Rightarrow \frac{dy}{dx} = (1+y^2) + (1+y^2)\frac{dy}{dx}$

$\Rightarrow y^2\frac{dy}{dx} = -(1+y^2)$

$\Rightarrow \frac{dy}{dx} = -\frac{1+y^2}{y^2}$ (proved)

Method - 2

$\sin(x+y) = y\cos(x+y)$

$\Rightarrow y = \tan(x+y)$

$\Rightarrow \tan^{-1} y = x+y$

$\Rightarrow \frac{1}{1+y^2}\cdot\frac{dy}{dx} = 1+\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}\left(\frac{1}{1+y^2}\right) = 1$

$\Rightarrow \frac{dy}{dx} = \frac{-(1+y^2)}{y^2}$

11 If $\sqrt{1-x^4}+\sqrt{1-y^4}=k(x^2-y^2)$

then show that

$$\frac{dy}{dx}=\frac{x\sqrt{1-y^4}}{y\sqrt{1-x^4}}$$

Ans ⇒ $\sqrt{1-x^4}+\sqrt{1-y^4}=k(x^2-y^2)$

$$\left[\begin{array}{l} \text{put } x^2=\cos\theta \\ y^2=\cos\theta \end{array}\right.$$

$\Rightarrow \sqrt{1-\cos^2\theta}+\sqrt{1-\cos^2\theta}$

$=k(\cos\theta - \cos\theta)$

$\Rightarrow \sin\theta+\sin\phi=k(\cos\theta-\cos\theta)$

$\Rightarrow 2\sin\frac{\theta+\phi}{2}\cdot\cos\frac{\theta-\phi}{2}$

$=k\cdot 2\sin\frac{\theta+\phi}{2}\cdot\sin\frac{\phi-\theta}{2}$

$\Rightarrow \cos\frac{\theta-\phi}{2}=k\sin\frac{\phi-\theta}{2}=-k\sin\frac{\theta-\phi}{2}$

$\Rightarrow \tan\frac{\theta-\phi}{2}=\frac{1}{-k}$

Differentiating both sides w.r.t x we get

$\frac{d}{dx}\left\{\tan\frac{\theta-\phi}{2}\right\}=0$ (∵ k is a constant)

$\sec^2\frac{\theta-\phi}{2}\times\frac{d}{dx}\frac{\theta-\phi}{2}=0$

$\Rightarrow$

$$\Rightarrow \frac{d\theta}{dx} - \frac{d\phi}{dx} = 0$$

$$\Rightarrow \frac{d}{dx}(\cos^{-1} x^2) - \frac{d}{dx}(\cos^{-1} y^2) = 0$$

$$\Rightarrow \frac{-1}{\sqrt{1-x^4}} \frac{d}{dx}(x^2) + \frac{1}{\sqrt{1-y^4}} \cdot \frac{d}{dx}(y^2) = 0$$

$$\Rightarrow -\frac{2x}{\sqrt{1-x^4}} + \frac{2y}{\sqrt{1-y^4}} \cdot \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{x\sqrt{1-y^4}}{y\sqrt{1-x^4}}$$