

Date $\rightarrow$ 25.10.23

INTEGRATION

Indefinite Integrals

$$(1) \int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$$

$$(2) \int \frac{1}{x} dx = \log_e |x| + C$$

$$(3) \int e^x dx = e^x + C$$

$$(4) \int a^x dx = \frac{a^x}{\log_e a} + C$$

$$(5) \int \sin x dx = -\cos x + C$$

$$(6) \int \cos x dx = \sin x + C$$

$$(7) \int \sec x dx = \tan x + C$$

$$(8) \int \csc x dx = -\cot x + C$$

$$(9) \int \sec x \tan x dx = \sec x + C$$

$$(10) \int \csc x \cot x dx = -\csc x + C$$

$$(11) \int \cot x dx = \log |\sin x| + C$$

$$(12) \int \tan x dx = -\log |\cos x| + C$$

$$(13) \int \sec x dx = \log |\sec x + \tan x| + C$$

$$(14) \int \csc x dx = \log \left| \tan \left(\frac{\pi}{4} + \frac{x}{2} \right) \right| + C$$

$$\int \csc x = \csc x - \cot x + C = \log \left| \tan \frac{x}{2} \right| + C$$

$$(15) \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \left(\frac{x}{a} \right) + C$$

$$(16) \int dx = x + C \quad (17) \int \frac{dx}{x} = \ln x + C$$

$$(16) \int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1} \left(\frac{x}{a} \right) + c$$

$$(17) \int \frac{1}{\sqrt{a^2 + x^2}} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c$$

$$(18) \int \frac{-1}{\sqrt{a^2 + x^2}} dx = \frac{1}{a} \cot^{-1} \frac{x}{a} + c$$

$$(19) \int \frac{1}{x \sqrt{x^2 - a^2}} dx = \frac{1}{a} \sec^{-1} \left(\frac{x}{a} \right) + c$$

$$(20) \int \frac{-1}{x \sqrt{a^2 - x^2}} dx = \frac{1}{a} \operatorname{cosec}^{-1} \left(\frac{x}{a} \right) + c$$

EXERCISE - 9 (a)

1/ (i) $\int x dx$

Ans $\int x dx = \frac{x^2}{2} + c$

(ii) $\int 3x^2 dx$

Ans $\int 3x^2 dx = 3 \frac{x^3}{3} + c = x^3 + c$

(iii) $\int x^3 dx$

Ans $\int x^3 dx = \frac{x^4}{4} + c = \frac{x^4}{4} + c$

(iv) $\int x^5 dx$

Ans $\int x^5 dx = \frac{x^6}{6} + c$

$$(v) \int x^3 dx$$

$$\text{Ans} \Rightarrow \int x^3 dx = \frac{x^4}{4} + C$$

$$(vi) \int \left(2x + \frac{3}{x} \right) dx$$

$$\text{Ans} \Rightarrow \int \left(2x + \frac{3}{x} \right) dx$$

$$= 2 \int x^{\frac{1}{2}} dx + 3 \int x^{-\frac{1}{2}} dx = 2 \cdot \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + 3 \cdot \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}$$

$$= 2 \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 3 \cdot \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C = \frac{4}{3} x^{\frac{3}{2}} + 6x^{\frac{1}{2}} + C$$

$$(vii) \int \frac{dx}{x^2} = \int x^{-2} dx$$

$$= \frac{x^{-2+1}}{-2+1} + C = \frac{x^{-1}}{-1} + C = -\frac{1}{x} + C$$

$$(viii) \int \left(x^{\frac{4}{7}} + \frac{1}{x^{\frac{1}{3}}} \right) dx = \int x^{\frac{4}{7}} dx + \int x^{-\frac{1}{3}} dx$$

$$= \frac{x^{\frac{4}{7}+1}}{\frac{4}{7}+1} + \frac{x^{-\frac{1}{3}+1}}{-\frac{1}{3}+1} + C = \frac{7}{11} x^{\frac{11}{7}} + \frac{3}{2} x^{\frac{2}{3}} + C$$

$$(ix) \int \frac{4}{x} dx = 4 \ln |x| + C$$

$$(x) \int \left(\frac{3}{x^2} + \frac{1}{x^{1/2}} \right) dx$$

$$= 3 \int x^{-2} dx + \int x^{-1/2} dx$$

$$= 3 \frac{x^{-1}}{-1} + \frac{x^{1/2}}{1/2} + C$$

$$= -\frac{3}{x} - \frac{1}{1/2} \cdot \frac{1}{x^{1/2}} + C$$

$$(xi) \int (x^2 + \sqrt{x})^2 dx$$

$$= \int (x^4 + x + 2x^2 \cdot \sqrt{x}) dx$$

$$= \int x^4 dx + \int x dx + 2 \int x^{\frac{5}{2}} dx$$

$$= \frac{1}{5} x^5 + \frac{1}{2} x^2 + 2 \times \frac{2}{\frac{5}{2} + 1} x^{\frac{5}{2} + 1} + C$$

$$= \frac{1}{5} x^5 + \frac{1}{2} x^2 + \frac{4}{7} x^{\frac{7}{2}} + C$$

$$(xii) \int (x+3)(2-x) dx$$

$$= \int (2x+6 - x^2 - 3x) dx$$

$$= \int (-x^2 - x + 6) dx$$

$$= -\frac{1}{3} x^3 - \frac{1}{2} x^2 + 6x + C$$

$$(xiii) \int \frac{(x+2)^2}{x^4} dx$$

$$= \int \frac{x^2 + 4x + 4}{x^4} dx$$

$$= \int (x^{-2} + 4x^{-3} + 4x^{-4}) dx$$

$$= \frac{x^{-2}}{-2} + 4 \frac{x^{-3}}{-3} + 4 \frac{x^{-4}}{-4} + C$$

$$= -\frac{1}{2x} - \frac{4}{3x^3} - \frac{1}{x^4} + C$$

$$(xiv) \int \frac{(x+\sqrt{x})(2x+1)}{x^2} dx$$

$$= \int \frac{2u^2 + x + 2x^{\frac{2}{3}} + x^{\frac{1}{3}}}{x^{\frac{2}{3}}} dx$$

$$= \int \left\{ 2 + \frac{1}{x} + 2x^{-\frac{1}{3}} + x^{-\frac{2}{3}} \right\} dx$$

$$= 2x + \ln |x| + \frac{2x^{\frac{1}{3}}}{\frac{1}{3}} + \frac{x^{-\frac{1}{3}}}{-\frac{1}{3}} + C$$

$$= 2x + \ln |x| + 6x^{\frac{1}{3}} - \frac{3}{x^{\frac{1}{3}}} + C$$

1/ $R = \{ (m, n) : \frac{m}{n} \text{ is a power of } 5 \}$

Soln

Reflexive :-

Let $m, n \in \mathbb{Z} - \{0\}$

$$\frac{m}{n} = \frac{m}{n} = 5^0$$

So, $(m, m) \in R$

So, R is reflexive

Symmetric

$m, n \in \mathbb{Z} - \{0\}$

Let $(m, n) \in R$

$$\Rightarrow \frac{m}{n} = 5^k$$

$$\Rightarrow \frac{n}{m} = \frac{1}{5^k} = 5^{-k}$$

$\therefore (n, m) \in R$

So, R is symmetric

Transitive :-

$m, n, p \in \mathbb{Z} - \{0\}$

Let $(m, n) \in R$ and $(n, p) \in R$

$$\Rightarrow \frac{m}{n} = 5^{k_1} \quad \text{--- (1)}$$

$$= \frac{7}{p} = 5k_2 \quad \text{--- (ii)}$$

$$\text{eqn (i)} \times \text{eqn (ii)}$$

$$\Rightarrow \frac{m}{x} \times \frac{x}{p} = 5k_1 \cdot 5k_2$$

$$\Rightarrow \frac{m}{p} = 5k_1 + k_2$$

$$(m, p) \in R$$

R is transitive

R is reflexive, symmetric and transitive so, R is an equivalence relation.

$$\Rightarrow \tan \left(\cos^{-1} \frac{4}{5} + \tan^{-1} \frac{2}{3} \right)$$

$$\text{Ans} \Rightarrow \text{Let } \cos^{-1} \frac{4}{5} = \alpha, \sin \alpha = \sqrt{1 - \cos^2 \alpha}$$

$$\Rightarrow \frac{4}{5} = \cos \alpha$$

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha} = \tan \alpha = \frac{3}{4}$$

$$= \frac{3}{4} \Rightarrow \alpha = \tan^{-1} \frac{3}{4}$$

$$\tan \left(\tan^{-1} \frac{3}{4} + \tan^{-1} \frac{2}{3} \right)$$

$$\tan \left(\frac{\tan^{-1} \frac{3}{4} + \tan^{-1} \frac{2}{3}}{1 - \frac{3}{4} \times \frac{2}{3}} \right)$$

$$= \tan \left(\tan^{-1} \frac{17}{12} \right) = \frac{17}{12}$$

$$\tan \left(\tan^{-1} \frac{17}{12} \right) = \frac{17}{12}$$

(3)

$$x + iy = z = x + iy$$

$$\text{subject to } x + y \leq 50 \quad \text{--- (i)}$$

$$x + 2y \leq 80 \quad \text{--- (ii)}$$

$$x + y \geq 20 \quad \text{--- (iii)}$$

$$x + y \geq 0$$

Ans. =

Step 1. Take

$$(i) \int \cos x \, dx$$

$$\text{Ans} \Rightarrow \int \cos x \, dx = \sin x + C$$

$$(ii) \int \frac{dx}{\cos x}$$

$$\text{Ans} \Rightarrow \int \frac{dx}{\cos x} = \int \sec x \, dx = \tan x + C$$

$$(iii) \int \frac{dx}{1 - \cos x}$$

$$\text{Ans} \Rightarrow \int \frac{dx}{1 - \cos x} = \int \frac{dx}{\sin^2 x} = \int \csc^2 x \, dx$$

$$= -\cot x + C$$

$$(iv) \int \frac{\sin x}{\cos x} \, dx = \int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} \, dx$$

$$= \int \sec x \cdot \tan x \, dx = \sec x + C$$

$$(v) \int \frac{2 \cos x}{1 - \cos x} \, dx = 2 \int \frac{\cos x}{1 - \cos x} \, dx$$

$$= 2 \int \frac{1}{\sin x} \cdot \frac{\cos x}{\sin x} \, dx$$

$$= 2 \int \csc x \cdot \cot x \, dx$$

$$= -2 \csc x + C$$

$$(vi) \int \frac{1 - \sin^3 x}{\sin^2 x} dx = \int (\operatorname{cosec}^2 x - \sin x) dx$$

$$= \left(\frac{1}{\sin^2 x} - \frac{\sin^3 x}{\sin^2 x} \right) dx$$

$$= -\cot x + \cos x + C$$

$$(vii) \int \frac{\sin^2 x}{1 + \cos x} dx = \int \frac{\sin^2 x (1 - \cos x)}{1 - \cos^2 x} dx$$

$$= \int \frac{\sin^2 x (1 - \cos x)}{\sin^2 x} dx$$

$$= \int (1 - \cos x) dx$$

$$= \int dx - \int \cos x dx = x - \sin x + C$$

$$(viii) \int \frac{\cos^2 x - \sin^2 x}{\cos x + \sin x} dx = \int \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x + \sin x} dx$$

$$= \int (\cos x - \sin x) dx = \sin x + \cos x + C$$

$$(ix) \int \frac{\cos^4 x - \sin^4 x}{\cos x - \sin x} dx$$

$$= \int \frac{(\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x)}{(\cos x - \sin x)} dx$$

$$= \int (\cos x + \sin x) dx = \sin x - \cos x + C$$

$$(x) \int \frac{\cos^2 x}{\sin^2 x \cdot \cos^2 x} dx = \int \left(\frac{\cos^2 x}{\sin^2 x} - \frac{\sin^2 x}{\sin^2 x \cos^2 x} \right) dx$$

$$= \int \frac{\cos^2 x - \sin^2 x}{\sin^2 x \cos^2 x} dx = \int \left(\frac{\cos^2 x}{\sin^2 x \cos^2 x} - \frac{\sin^2 x}{\sin^2 x \cos^2 x} \right) dx$$

$$= \int (\operatorname{cosec}^2 x - \sec^2 x) dx$$

$$= -\cot x - \tan x + C$$

(vi) $\int \frac{a^2 \sin^2 x + b^2 \cos^2 x}{\sin^2 x \cos^2 x} dx$

$$= \int \frac{a^2 \sin^2 x + b^2 \cos^2 x}{\sin^2 x \cos^2 x} dx$$

$$= \frac{a^2}{4} \int \sec^2 x dx + \frac{b^2}{4} \int \operatorname{cosec}^2 x dx$$

$$= \frac{a^2}{4} \tan x - \frac{b^2}{4} \cot x + C$$

(vii) $\int (\tan x + \cot x)^2 dx$

$$= \int \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right)^2 dx$$

$$= \int \left(\frac{\sin^2 x + \cos^2 x}{\cos x \sin x} \right)^2 dx$$

$$= \int \frac{1}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx$$

$$= \int (\sec^2 x + \operatorname{cosec}^2 x) dx = \tan x - \cot x + C$$

(viii) $\int \frac{1 - \cos 2x}{1 + \cos 2x} dx = \int \frac{2 \sin^2 x}{2 \cos^2 x} dx$

$$= \int \tan x dx = \int (\sec x - 1) dx$$

$$= \tan x - x + C$$

(vii) $\int \sec x \cdot \operatorname{cosec} x dx$

$$= \int \frac{1}{\sin x \cos x} dx$$

$$= \int \frac{\sin x + \cos x}{\sin x \cdot \cos x} dx$$

$$= \int (\sec x + \operatorname{cosec} x) dx$$

$$= \tan x - \cot x + C$$

(viii) $\int \frac{a \sin^3 x + b \cos^3 x}{\sin x \cos x} dx$

$$= \int \left\{ \frac{a \sin^3 x}{\sin x \cos x} + \frac{b \cos^3 x}{\sin x \cos x} \right\} dx$$

$$= \int \left\{ a \frac{\sin^2 x}{\cos x} + b \frac{\cos^2 x}{\sin x} \right\} dx$$

$$= \int (a \sec x \cdot \tan x + b \operatorname{cosec} x \cdot \cot x) dx$$

$$= a \int \sec x \cdot \tan x dx + b \int \operatorname{cosec} x \cdot \cot x dx$$

$$= a \sec x - b \operatorname{cosec} x + C$$

(ix) $\int \sqrt{1 + \sin x} dx$

$$= \int \sqrt{\cos^2 \frac{x}{2} + \sin^2 \frac{x}{2} + 2 \sin \frac{x}{2} \cos \frac{x}{2}} dx$$

$$= \int \sqrt{(\cos x + \sin x)^2} dx$$

$$= \int (\cos x + \sin x) dx$$

$$= \sin x - \cos x + C$$

$$(vii) \int \sqrt{1 - \cos 2x} dx = \int \sqrt{2} \sin x dx$$

$$= \sqrt{2} \int \sin x dx = -\sqrt{2} \cos x + C$$

$$(viii) \int \sqrt{1 + \cos 2x} dx = \int \sqrt{2} \cos x dx$$

$$= \int \sqrt{2} \cos x dx$$

$$= \sqrt{2} \sin x + C$$

$$(ix) \int \frac{\cos x \cdot \cos 2x + \sin x \cdot \sin 2x}{1 - \cos 2x} dx$$

$$= \int \frac{\cos(x - 2x)}{1 - \cos 2x} dx$$

$$= \int \frac{\cos x}{\sin x} dx = \int \cot x dx = \log |\sin x| + C$$

$$= \int \csc x \cdot \cot x dx = -\csc x + C$$

$$(x) \int (a \cot x + b \tan x) dx$$

$$= \int \{a \cot x + b \tan x\} dx$$

$$= \int (a \cot x + b \tan x) dx$$

$$= a \int \cot x dx + b \int \tan x dx$$

$$= a \int \frac{\cos x}{\sin x} dx + b \int \frac{\sin x}{\cos x} dx$$

$$= a \int \csc x dx + b \int \sec x dx$$

$$= a \log |\csc x - 1| + b \log |\sec x - 1| + C$$

$$\begin{aligned}
 &= a^2(-\cot u - u) + b^2(\tan u - u) + 2ab \cot u \\
 &= b^2 \tan u - a^2 \cot u - u(a^2 + b^2) - 2ab \cot u \\
 &= b^2 \tan u - a^2 \cot u - (ab)^2 \cot u \\
 &= b^2 \tan u - a^2 \cot u + C
 \end{aligned}$$

$$\begin{aligned}
 \text{2) (i)} \quad \int (e^x + 2) dx &= \int e^x dx + 2 \int dx \\
 &= e^x + 2x + C
 \end{aligned}$$

$$\text{(ii)} \quad \int 3^x dx = \frac{3^x}{\ln 3} + C$$

$$\begin{aligned}
 \text{(iii)} \quad \int a^{x+2} dx &= \int a^2 a^x dx \\
 &= a^2 \cdot \frac{a^x}{\ln a} + C = \frac{a^{x+2}}{\ln a} + C
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad \int a^{2x} dx &= \frac{1}{2} \frac{a^{2x}}{\ln a} + C \\
 \text{(v)} \quad \int \frac{e^{2x} + 1}{e^x} dx &= \int (e^x + e^{-x}) dx \\
 &= \frac{1}{2} \int a^t \cdot \frac{dt}{a} \\
 &= \frac{1}{2} \int a^t dt \\
 &= \frac{1}{2} \frac{a^t}{\ln a} \\
 &= \frac{a^{2x} + 1}{2 \ln a} + C
 \end{aligned}$$

$$= e^x - e^{-x} + C$$

$$\frac{28 \cdot 10 \cdot 23}{2}$$

$$\text{4) (i)} \quad \int \left(\frac{5}{\sqrt{1-x^2}} + \frac{7}{1+x^2} \right) dx$$

$$= 5 \int \frac{dx}{\sqrt{1-x^2}} + 7 \int \frac{dx}{1+x^2}$$

$$= 5 \sin^{-1} x + 7 \tan^{-1} x + C$$

$$(i) \int \frac{3x^2}{x^2+1} dx$$

$$= 3 \int \frac{(x^2+1)-1}{x^2+1} dx$$

$$= 3 \int \left\{ 1 - \frac{1}{x^2+1} \right\} dx$$

$$= 3 \left\{ x - \tan^{-1} x \right\} + C$$

$$(iii) \int \frac{x^8}{x^2+1} dx = \int \frac{(x^2+1)-1}{x^2+1} dx$$

$$= \int \frac{x^6+1}{x^2+1} dx = \int \frac{x^6}{x^2+1} dx + \int \frac{1}{x^2+1} dx$$

$$= \int \frac{x^3 \cdot x^3}{x^2+1} dx + \int \frac{1}{x^2+1} dx$$

$$= \int \frac{(x^2+1)(x^4-x^2)}{x^2+1} dx + \int \frac{1}{x^2+1} dx$$

$$= \int (x^4 - x^2) dx + \int \frac{1}{1+x^2} dx$$

$$= \frac{1}{5} x^5 - \frac{1}{3} x^3 + x - \tan^{-1} x + C$$

$$(iv) \int \frac{x^4 + x^3 + x^2 + x + 2}{x^2+1} dx$$

$$= \int \frac{(x^4 + x^2) + (x^3 + x) + 2}{x^2+1} dx$$

$$= \int \frac{x^2(x^2+1) + x(x^2+1) + 2}{x^2+1} dx$$

$$= \int \left\{ x^2 + x + \frac{2}{x^2 + 2} \right\} dx = \int \frac{(x^2 + 1)(x^2 + 2) + 2}{(x^2 + 1)(x^2 + 2)} dx$$

$$= \frac{1}{3} x^3 + \frac{1}{2} x^2 + 2 \tan^{-1} x + C$$

$$(v) \int \left\{ \sqrt{1-x^2} + \frac{x^2}{\sqrt{1-x^2}} \right\} dx$$

$$= \int \left\{ \frac{1-x^2+x^2}{\sqrt{1-x^2}} \right\} dx = \int \frac{dx}{\sqrt{1-x^2}}$$

$$= \sin^{-1} x + C$$

$$(vi) \int \frac{x^2 + \sqrt{x^2 - 1}}{x^3 \sqrt{x^2 - 1}} dx$$

$$= \int \left\{ \frac{1}{x \sqrt{x^2 - 1}} + \frac{1}{x^3} \right\} dx$$

$$= \int \frac{dx}{x \sqrt{x^2 - 1}} + \int x^{-3} dx$$

$$= \sec^{-1} x + \frac{x^{-3+1}}{-3+1} + C$$

$$= \sec^{-1} x = \frac{1}{x^2} + C$$

Antiderivative = Integration

$$(5) \text{ Ans} \rightarrow \int f(x) dx = \int (2x^2 + 1) dx$$

$$= \frac{2}{3} x^3 + x + C$$

$$\text{But } f(x) = -2$$

$$\therefore -2 = C$$

$$\text{Thus } f(x) = \frac{2}{3} x^3 + x - 2$$