EXERCISE - 9(f) Date - 23.11.23

1/(i) $\int \frac{4x-9}{x^2-5x+6}\,dx$

$= \int \frac{4x-9}{x^2-3x-2x+6}\,dx$

$= \int \frac{4x-9}{x(x-3)-2(x-3)}\,dx$

$= \int \frac{\cancel{x}\,4x-9}{(x-3)(x-2)}\,dx$

$\frac{4x-9}{(x-3)(x-2)} = \frac{A}{x-3} + \frac{B}{x-2}$

$\frac{4x-9}{(x-3)(x-2)} = \frac{A(x-2)+B(x-3)}{(x-3)(x-2)}$

$4x-9 = A(x-2) + B(x-3)$

$x=2 \Rightarrow 4\cdot 2-9 = A(2-2) + B(2-3)$

$\Rightarrow -1 = -B$

$\Rightarrow B = 1$

$x=3 \Rightarrow 4\cdot 3-9 = A(3-2) + B(3-3)$

$\Rightarrow 3 = A$

$\Rightarrow A = 3$

$$\therefore \int \frac{4x-9}{x^2-5x+6}\,dx = \int \frac{3}{x-3}\,dx + \int \frac{1}{x-2}\,dx$$

$$= 3\ln(x-3) + \ln(x-2) + c$$

$$= \ln\{(x-3)^3 (x-2)\} + c$$

(ii) $\int \frac{3x}{(x-4)(x+2)}\,dx$

$$\frac{3x}{(x-4)(x+2)} = \frac{A}{(x-4)} + \frac{B}{(x+2)}$$

$$\frac{3x}{(x-4)(x+2)} = \frac{A(x+2) + B(x-4)}{(x-4)(x+2)}$$

$$3x = A(x+2) + B(x-4)$$

$$x=-2 \Rightarrow 3\cdot -2 = A(-2+2) + B(-2-4)$$

$$\Rightarrow -6 = B(-6)$$

$$\Rightarrow B = 1$$

$$x = 4 \Rightarrow 3\cdot 4 = A(4+2) + B(4-4)$$

$$\Rightarrow 12 = A6$$

$$\Rightarrow A = \cancel{\frac{12}{6}}\ 2$$

$$\therefore \int \frac{3x}{(x-4)(x+2)}\,dx = \int \frac{2}{x-4}\,dx + \int \frac{1}{x+2}\,dx$$

$$= 2\ln(x-4) + \ln(x+2) + c$$

$$= \ln(x-4)^2(x+2) + c$$

(iii) $\int \frac{5x-12}{(2x-3)(x-6)}\,dx$

$$\frac{5x-12}{(2x-3)(x-6)} = \frac{A}{2x-3} + \frac{B}{x-6}$$

$$\frac{5x-12}{(2x-3)(x-6)} = \frac{A(x-6)+B(2x-3)}{(2x-3)(x-6)}$$

$$5x-12 = A(x-6)+B(2x-3)$$

$$x=6 \Rightarrow 5\cdot 6-12 = A(6-6)+B(2\cdot 6-3)$$

$$18 = B(9)$$

$$B = \frac{18}{9} = 2$$

$$x=\frac{3}{2} \Rightarrow 5\cdot\frac{3}{2}-12 = A\left(\frac{3}{2}-6\right)+B\left(2\cdot\frac{3}{2}-3\right)$$

$$\Rightarrow \frac{15-24}{2} = A\frac{(3-12)}{2}$$

$$\Rightarrow -\frac{9}{2} = A\cdot -\frac{9}{2}$$

$$\Rightarrow A = 1$$

(iii)

$$\int \frac{5x-12}{(2x-3)(x-6)}\,dx = \int \frac{1}{2x-3}\,dx + \int \frac{2}{x-6}\,dx$$

$$= \frac{1}{2}\ln(2x-3) + 2\ln(x-6)+c$$

$$= \ln\sqrt{2x-3}\,(x-6)^2+c$$

(iv) $\int \frac{20x+3}{6x^2+x-2}\,dx$

$$= \int \frac{20x+3}{6x^2-4x+3x-2}\,dx$$

$$= \int \frac{20x+3}{2x(3x-2)+1(3x-2)}\,dx$$

$$= \int \frac{20x+3}{(2x+1)(3x-2)}\,dx$$

$$\frac{20x+3}{(2x+1)(3x-2)} = \frac{A}{2x+1} + \frac{B}{3x-2}$$

$$\frac{20x+3}{(2x+1)(3x-2)} = \frac{A(3x-2)+B(2x+1)}{(2x+1)(3x-2)}$$

$$20x+3 = A(3x-2) + B(2x+1)$$

$$x = \frac{2}{3} \Rightarrow 20\cdot\frac{2}{3}+3 = A\left(3\cdot\frac{2}{3}-2\right)+B\left(2\cdot\frac{2}{3}+1\right)$$

$$\Rightarrow \frac{40}{3}+3 = A\cdot 0 + B\left(\frac{4}{3}+1\right)$$

$$\Rightarrow \frac{49}{3} = B\,\frac{7}{3}$$

$$\Rightarrow B = \frac{\cancel{49}^{\,7}}{\cancel{3}}\cdot\frac{\cancel{3}}{\cancel{7}}$$

$$\Rightarrow B = 7$$

$$x = -\frac{1}{2} \Rightarrow \cancel{20}^{\,10}\cdot -\frac{1}{\cancel{2}} + 3 = A\left(3\cdot\frac{-1}{2}-2\right) + B\left(2\cdot -\frac{1}{2}+1\right)$$

$$\Rightarrow -10+3 = A\left(\frac{-3-4}{2}\right)$$

$\Rightarrow -7 = A\left(\frac{-7}{2}\right)$

$\Rightarrow A = \frac{\cancel{7}}{\frac{\cancel{7}}{2}}$

$\Rightarrow A = 2$

$\therefore \int \frac{20x+3}{6x^2-x-2}\,dx = \int \frac{2}{(2x+1)}\,dx + \int \frac{7}{3x-2}\,dx$

$= \frac{1}{2}\ln(2x+1) + 7\cdot\frac{1}{3}\ln(3x-2) + C$

$= \ln(2x+1) + \frac{7}{3}\ln(3x-2) + C$

$= \ln\left\{(2x+1)\cdot(3x-2)^{\frac{7}{3}}\right\} + C$

(v) $\int \frac{2x^2}{(x-1)(x-2)(x-3)}\,dx$

$\frac{2x^2}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$

$= \frac{A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)}{(x-1)(x-2)(x-3)}$

$\frac{2x^2}{(x-1)(x-2)(x-3)} = \frac{A(x^2-5x+6) + B(x^2-4x+3) + C(x^2-3x+2)}{(x-1)(x-2)(x-3)}$

$2x^2 = A(x^2-5x+6) + B(x^2-4x+3) + C(x^2-3x+2)$

~~Equal~~ Equating the coefficient of x^2 in both side $2 = A+B+C$

$2x^2 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$

$x=2 \Rightarrow 2\cdot(2)^2 = A(2-2)(2-3) + B(2-1)(2-3) + C(2-1)(2-2)$

$\Rightarrow 8 = B(-1)$

$\Rightarrow B = -8$

$x=3 \Rightarrow 2\cdot(3)^2 = A(3-2)(3-3) + B(3-1)(3-3) + C(3-1)(3-2)$

$\Rightarrow 18 = C(2)$

$\Rightarrow C = 9$

$x=1 \Rightarrow 2\cdot(1)^2 = A(1-2)(1-3) + B(1-1)(1-3) + C(1-1)(1-2)$

$\Rightarrow 2 = A(2)$

$\Rightarrow A = 1$

$\therefore$

$$\int \frac{2x^2}{(x-1)(x-2)(x-3)}dx = \int \frac{1}{x-1} + \int \frac{-8}{x-2}dx + \int \frac{9}{x-3}dx$$

$$= \ln(x-1) - 8\ln(x-2) + 9\ln(x-3) + c$$

(4)

$$\int \frac{12x^4 - 2x^3 - 4x^2 + 4x - 3}{6x^2 - x - 2}dx$$

$2x^2$

$6x^2-x-2 \,)\, 12x^4 - 2x^3 - 4x^2 + 4x - 3$

$12x^4 - 2x^3 - 4x^2$

$- \quad + \quad +$

$x - 3$

$$= \int 2x^2 + \frac{x-3}{6x^2-x-2}\,dx$$

$$= \int 2x^2\,dx + \int \frac{x-3}{6x^2-x-2}\,dx$$

$$= 2\frac{x^3}{3} + \int \frac{x-3}{6x^2-4x+3x-2}\,dx$$

$$= \frac{2x^3}{3} + \int \frac{x-3}{2x(3x-2)+1(3x-2)}\,dx$$

$$= \frac{2}{3}x^3 + \int \frac{x-3}{(2x+1)(3x-2)}\,dx$$

$$= \frac{2}{3}x^3 + \int \frac{x-3}{(2x+1)(3x-2)}\,dx$$

$$= \frac{2}{3}x^3 + I$$

$$I = \frac{x-3}{(2x+1)(3x-2)} = \frac{A}{2x+1} + \frac{B}{3x-2}$$

$$\Rightarrow \frac{x-3}{(2x+1)(3x-2)} = \frac{A(3x-2)+B(2x+1)}{(2x+1)(3x-2)}$$

$$\Rightarrow x-3 = A(3x-2) + B(2x+1)$$

$$x = -\frac{1}{2} \Rightarrow -\frac{1}{2} - 3 = A\left(3\cdot\frac{-1}{2} - 2\right) + B\left(2\cdot\frac{-1}{2} + 1\right)$$

$$\Rightarrow \frac{-1-6}{2} = A\left(\frac{-3-4}{2}\right) + 0$$

$$\Rightarrow -\frac{7}{2} = A\left(\frac{-7}{2}\right)$$

$$\Rightarrow A = 1$$

$x = \frac{2}{3} \Rightarrow \frac{2}{3} - 3 = A\left(3 \cdot \frac{2}{3} - 2\right) + B\left(2 \cdot \frac{2}{3} + 1\right)$

$\Rightarrow \frac{2-9}{3} = +B\left(\frac{4+3}{3}\right)$

$\Rightarrow \frac{-7}{3} = B\frac{7}{3}$

$\Rightarrow B = -1$

$\therefore \int \frac{12x^4 - 2x^3 - 4x^2 + x - 3}{6x^2 - x - 2}dx = \frac{2}{3}x^3 + \int \frac{7}{2x+1}dx + \int \frac{-1}{3x-2}dx$

$= \frac{2}{3}x^3 + \frac{7}{2}\ln(2x+1) - \frac{1}{3}\ln(3x-2) + C$

2(i) $\int \frac{2x+9}{(x+3)^2}dx$

$\frac{2x+9}{(x+3)^2} = \frac{A}{(x+3)} + \frac{B}{(x+3)^2}$

$= \frac{A(x+3) + B}{(x+3)^2}$

$2x + 9 = A(x+3) + B$

$x = -3 \Rightarrow 2(-3) + 9 = A(-3+3) + B$

$\Rightarrow -6 + 9 = B \Rightarrow B = 3$

$2x + 9 = A(x+3) + 3$

$A(x+3) + 3 = 2x + 9$

$A(x+3) = 2x+9-3$

$A = \dfrac{2x+6}{x+3}$

$= \dfrac{2(x+3)}{x+3}$

$A = 2$

$\therefore \int \dfrac{2x+9}{(x+3)^2}\,dx = \int \dfrac{2}{x+3}\,dx + \int \dfrac{3}{(x+3)^2}\,dx$

$= 2\ln(x+3) + 3 \cdot -1(x+3)^{-1}$

$= 2\ln(x+3) - \dfrac{3}{(x+3)} + c$

(ii) $\int \dfrac{5x^2+4x+4}{(x-2)(x+2)^2}\,dx$

(ii) Let $\dfrac{5x^2+4x+4}{(x-2)(x+2)^2} = \dfrac{A}{x-2} + \dfrac{B}{x+2} + \dfrac{C}{(x+2)^2}$

Then $A(x+2)^2 + B(x-2)(x+2) + C(x-2) = 5x^2+4x+4$ —(1)

Put $x = -2$

$A(-2+2)^2 + B(-2-2)(-2+2) + C(-2-2) = 5\times(-2)^2 + 4(-2) + 4$

$\Rightarrow A\times 0 + B\times 0 + C(-4) = 20-8+4$

$\Rightarrow -4C = 16$

$\Rightarrow C = \dfrac{16}{-4} = -4$

$x = 2$

$A(2+2)^2 + B(2-2)(2+2) + C(2-2) = 5(2)^2 + 4(2) + 4$

$\Rightarrow A \times (4)^2 + B \times 0 + C \times 0 = 5 \times 4 + 8 + 4$

$\Rightarrow 16A = 32$

$\Rightarrow A = \frac{32}{16} = 2$

Again, eqn (i) becomes

$A(x+2)^2 + B(x-2)(x+2) + C(x-2) = 5x^2 + 4x + 4$

$\Rightarrow A(x^2 + 4x + 4) + B(x^2 - 4) + C(x-2) = 5x^2 + 4x + 4$

$\Rightarrow Ax^2 + A4x + A4 + Bx^2 - B4 + Cx - C2 = 5x^2 + 4x + 4$

equating the coefficient of x^2

$\Rightarrow A + B = 5$

$\Rightarrow B = 5 - A$
$= 5 - 2$
$= 3$

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(iii) $\int \frac{x^2+7x+4}{x^3+x^2-x-1}\,dx$

$= \int \frac{x^2+7x+4}{x^2(x+1)-1(x+1)}\,dx$

$= \int \frac{x^2+7x+4}{(x+1)(x^2-1)}\,dx = \int \frac{x^2+7x+4}{(x+1)(x+1)(x-1)}\,dx$

$= \int \frac{x^2+7x+4}{(x-1)(x+1)^2}$

$\frac{x^2+7x+4}{(x-1)(x+1)^2} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

$\frac{x^2+7x+4}{(x-1)(x+1)^2} = \frac{A(x+1)^2 + B(x-1)(x+1) + C(x-1)}{(x-1)(x+1)^2}$

$x^2+7x+4 = A(x+1)^2 + B(x-1)(x+1) + C(x-1)$ — (1)

$x = 1 \Rightarrow 1+7+4 = A(1+1)^2 + B(1-1)(1+1) + C(1-1)$

$\Rightarrow 12 = A(4) + 0 + 0$

$\Rightarrow A = 3$

$x = -1 \Rightarrow 1-7+4 = A(-1+1)^2 + B(-1-1)(-1+1) + C(-1-1)$

$\Rightarrow -2 = C(-2)$

$\Rightarrow C = 1$

equating the coefficient of x^2 in both side of eqn(1)

$1 = A + B$

$A + B = 1$

$3 + B = 1$

$B = 1 - 3 = -2$

$$\int \frac{x^4 + 3x^3 + x^2 - 1}{(x-1)(x+1)^2}\,dx = \int \frac{3}{x-1}\,dx \int \frac{-2}{x+1}\,dx + \int \frac{1\,dx}{(x+1)^2}$$

$$= 3\ln(x-1) - 2\ln(x+1) + (-1)(x+1)^{-1} + C$$

$$= 3\ln(x-1) - 2\ln(x+1) - \frac{1}{(x+1)} + C$$

2(i) $$\int \frac{4x^2 - x + 3}{(x^2+1)(x-1)}\,dx$$

$$\frac{4x^2 - x + 3}{(x^2+1)(x-1)} = \frac{A}{(x^2+1)} + \frac{B}{(x-1)}$$

$$\frac{4x^2 - x + 3}{(x^2+1)(x-1)} = \frac{A(x-1) + B(x^2+1)}{(x^2+1)(x-1)}$$

$$4x^2 - x + 3 = A(x-1) + B(x^2+1) \quad \text{--- (1)}$$

$x = 1 \Rightarrow 4(1)^2 - 1 + 3 = A(1-1) + B(1^2+1)$

$\Rightarrow 4 - 1 + 3 = B(2)$

$\Rightarrow 6 = B(2)$

$\Rightarrow B = 3$

Equating the co-efficient of x in both side in eqn (1)

$-1 = A + 0$

$A = -1$

$$\int \frac{4x^2 - x + 3}{(x^2+1)(x-1)}dx = \int \frac{-1}{(x^2+1)}dx + \int \frac{3}{x-1}dx$$

$$= -\frac{1}{2}\ln(x^2+1) + 3\ln(x+1)$$

(iii) $\left[\dfrac{5x}{(x^2-2x+2)(x+1)}dx\right]$

$= \dfrac{5x}{}$

(ii) $\displaystyle\int \frac{dx}{x^2-5}$

$\displaystyle\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + c$

$\displaystyle\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + c$

$= \displaystyle\int \frac{dx}{x^2-(\sqrt{5})^2}$

$= \dfrac{1}{2\sqrt{5}}\ln\left|\dfrac{x-\sqrt{5}}{x+\sqrt{5}}\right| + c$

$= \dfrac{1}{2\sqrt{5}}\ln\left|\dfrac{x-\sqrt{5}}{x+\sqrt{5}}\right| + c$

(ii) $\displaystyle\int \frac{dx}{2x^2+8x+7}$

$= \displaystyle\int \frac{dx}{2\left(x^2+4x+\frac{7}{2}\right)}$

$= \displaystyle\int \frac{dx}{2\left(x^2+4x+\frac{7}{2}\right)}$

$$= \frac{1}{2}\int \frac{dx}{(x)^2 + 2\cdot x\cdot 2 + (2)^2 - (2)^2 + \frac{7}{2}}$$

$$= \frac{1}{2}\int \frac{dx}{(x+2)^2 - 4 + \frac{7}{2}}$$

$$= \frac{1}{2}\int \frac{dx}{(x+2)^2 - \frac{1}{2}}$$

Let $x+2 = t$

$1 = \frac{dt}{dx}$

$dx = dt$

$$= \frac{1}{2}\int \frac{dt}{t^2 - \left(\frac{1}{\sqrt{2}}\right)^2}$$

$$= \frac{1}{2}\left[\frac{1}{2\cdot\frac{1}{\sqrt{2}}} \ln\left|\frac{t - \frac{1}{\sqrt{2}}}{t + \frac{1}{\sqrt{2}}}\right|\right] + c$$

$$= \frac{1}{2}\left[\frac{1}{\sqrt{2}} \ln\left|\frac{x+2 - \frac{1}{\sqrt{2}}}{x+2 + \frac{1}{\sqrt{2}}}\right|\right] + c$$

$$= \frac{1}{2\sqrt{2}} \ln\left|\frac{\sqrt{2}(x+2) - 1}{\sqrt{2}(x+2) + 1}\right| + c$$

4(iii)

$$\int \frac{(x+3)\,dx}{2x^2+8x+7}$$

Let $2x^2+8x+7 = t$

$4x+8 = \frac{dt}{dx}$

$4(x+2) = \frac{dt}{dx}$

$(x+2)dx = \frac{dt}{4}$

$$= \int \frac{x+2+1}{2x^2+8x+7}\,dx$$

$$= \int \frac{x+2}{2x^2+8x+7}\,dx + \int \frac{1}{2x^2+8x+7}\,dx$$

$$= \int \frac{dt}{4t} + \int \frac{dx}{2x^2+8x+7}$$

$$= \frac{1}{4}\ln t + \int \frac{dx}{2x^2+8x+7}$$

$= \frac{1}{4}\ln(2x^2+8x+7) + \frac{1}{4\sqrt{2}} \ln\left|\frac{\sqrt{2}x+2\sqrt{2}-1}{\sqrt{2}x+2\sqrt{2}+1}\right|$

(iv) $\int \frac{4x^2+20x+25}{2x^2+8x+7}\,dx$

$2x^2+7 \,)\, 4x^2+20x+25 \,(\, 2$
$4x^2+16x+14$
$4x+11$

$=\int\left(2+\frac{4x+11}{2x^2+8x+7}\right)dx$

$=\int 2\,dx + \int\frac{4x+8}{2x^2+8x+7}\,dx + \int\frac{3}{2x^2+8x+7}\,dx$

$= 2x+\int\frac{dt}{t}+3\int\frac{dt}{2x^2+8x+7}$

Let $2x^2+8x+7 = t$
$4x+8 = \frac{dt}{dx}$, $(4x+8)dx = dt$

$= 2x+\ln(2x^2+8x+7)+\frac{3}{2\sqrt{2}}\ln\left|\frac{\sqrt{2}(x+2)-1}{\sqrt{2}(x+2)+1}\right|+C$

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(v) $\int\frac{e^x}{e^{2x}+3e^x+1}\,dx$

Let $e^x=t$
$e^x=\frac{dt}{dx}$
$e^x dx=dt$

$=\int\frac{dt}{(t)^2+3t+1}$

$=\int\frac{dt}{t^2+2\cdot t\cdot\frac{3}{2}+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2+1}$

$=\int\frac{dt}{\left(t+\frac{3}{2}\right)^2-\frac{9}{4}+1}$

Let $t+\frac{3}{2}=z$
$1=\frac{dz}{dt}$
$dt=dz$

$=\int\frac{dt}{\left(t+\frac{3}{2}\right)^2-\frac{5}{4}}$

$=\int\frac{dz}{z^2-\left(\frac{\sqrt{5}}{2}\right)^2}$

$$= \frac{1}{2 \cdot \frac{\sqrt{5}}{2}} \ln \left| \frac{z - \frac{\sqrt{5}}{2}}{z + \frac{\sqrt{5}}{2}} \right| + C$$

$$= \frac{1}{\sqrt{5}} \ln \left| \frac{2\left(t + \frac{3}{2}\right) - \sqrt{5}}{2\left(t + \frac{3}{2}\right) + \sqrt{5}} \right| + C$$

$$= \frac{1}{\sqrt{5}} \ln \left| \frac{2e^{x} + 3 - \sqrt{5}}{2e^{x} + 3 + \sqrt{5}} \right| + C$$

(vi) $\displaystyle\int \frac{\tan^2\theta + 1}{\tan^2\theta - 1} d\theta$

Let $\tan\theta = t$

$\sec^2\theta = \frac{dt}{d\theta}$

$\sec^2\theta \, d\theta = dt$

$$= \int \frac{\sec^2\theta \, d\theta}{t^2 - 1}$$

$$= \int \frac{dt}{t^2 - 1}$$

$$= \frac{1}{2} \ln \left| \frac{t - 1}{t + 1} \right| + C$$

$$= \frac{1}{2} \ln \left| \frac{\tan\theta - 1}{\tan\theta + 1} \right| + C$$

5(i) $\displaystyle\int \frac{dx}{3 - x^2} = \int \frac{dx}{(\sqrt{3})^2 - x^2}$

$$= \frac{1}{2\sqrt{3}} \ln \left| \frac{\sqrt{3} + x}{\sqrt{3} - x} \right| + C$$

(ii) $\displaystyle\int \frac{dx}{7-x^2+6x}$

$= \displaystyle\int \frac{dx}{-(x^2-6x-7)}$

$= \displaystyle\int \frac{dx}{-(x^2-2\cdot x\cdot 3+(3)^2-(3)^2-7)} = \int \frac{dx}{-(x^2-2\cdot x\cdot 3+(3)^2-9-7)}$

$= -\displaystyle\int \frac{dx}{(x-3)^2-(4)^2} = \int \frac{dx}{x^2-2\cdot x\cdot 3+(3)^2-16}$

$= -\dfrac{1}{8}\ln\left|\dfrac{x-3+4}{x-3-4}\right| + c$

$= -\dfrac{1}{8}\ln\left|\dfrac{x+1}{x-7}\right| + c$

(iii) $\displaystyle\int \frac{x-5}{7-x^2+6x}\,dx$

$= \displaystyle\int \frac{x-5}{-(x^2-6x-7)}\,dx$

$= \displaystyle\int \frac{x-5}{-(x^2-2\cdot 3\cdot x+(3)^2-(3)^2-7}$

$= \displaystyle\int \frac{x-5\quad dx}{-(x^2-2\cdot 3\cdot x+(3)^2-(3)^2-7}$

$= \dfrac{x-5\; dx}{-\{(x-3)^2-(4)^2\}}$

$= \displaystyle\int \frac{x-5}{(4)^2-(x-3)^2}\,dx$

$= \displaystyle\int \frac{x-5}{(4+x-3)(4-x+3)}\,dx = \int \frac{x-5}{(x+1)(7-x)}\,dx$

$$\frac{x-5}{(x+1)(7-x)} = \frac{A}{x+1} + \frac{B}{7-x}$$

$$\frac{x-5}{(x+1)(7-x)} = \frac{A(7-x)+B(x+1)}{(x+1)(7-x)}$$

(iv) $\int \frac{\cos\theta}{3-\sin^2\theta} d\theta$

let $\sin\theta = t$

$\cos\theta = \frac{dt}{d\theta}$

$\cos\theta \, d\theta = dt$

$= \int \frac{dt}{(\sqrt{3})^2 - t^2}$

$= \frac{1}{2\sqrt{3}} \ln \left| \frac{t+\sqrt{3}}{t-\sqrt{3}} \right| + c$

$= \frac{1}{2\sqrt{3}} \ln \left| \frac{\sqrt{3}+\sin\theta}{\sqrt{3}-\sin\theta} \right| + c$

$= \frac{1}{2\sqrt{3}} \ln \left| \frac{\sqrt{3}+\sin\theta}{\sqrt{3}-\sin\theta} \right| + c$

(v) $\frac{x^2 \, dx}{x^2+3)}$

2(iv)

$\frac{x^4+3x^3+x^2-1}{x^3+x^2-x-1}$ $\cancel{(x+2)}$ $+ \frac{3x+1}{(x+1)^2(x-1)}$

~~Let~~ $\frac{3x+1}{(x-1)(x+1)^2} =$

Q (iv) $\int \frac{x^4+3x^3+x^2-1}{x^3+x^2-x-1}\,dx$

~~x^3+x^2-x-1~~

~~$x+2 + \frac{3x+x\cdot 1}{}$~~

$$
\begin{array}{r|l}
x^3+x^2-x-1 & x^4+3x^3+x^2-1 \;(x+2 \\
 & x^4+x^3-x^2-x \\
 & (-)\;(-)\;(+)\;(+) \\
\hline
 & 2x^3+2x^2+x-1 \\
 & 2x^3+2x^2-2x-2 \\
 & (-)\;(-)\;(+)\;(+) \\
\hline
 & 3x+1
\end{array}
$$

x^3+x^2-x-1
$\Rightarrow x^2(x+1)-1(x+1)$

(iv) $\int \frac{x^4+3x^3+x^2-1}{x^3+x^2-x-1} = (x+2) + \frac{3x+1}{(x+1)^2(x-1)}$

Let $\frac{3x+1}{(x-1)(x+1)^2} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

Then $A(x+1)^2 + B(x-1)(x+1) + C(x-1) = 3x+1$ ——— (1)

putting $x = 1, -1$

we get $4A = 3$ and $-2C = -2$

$\Rightarrow A = 1$ and $C = 1$

Again equating the co-efficients of x^2 on both sides of (1) we get

$A + B = 0$

$\Rightarrow B = -A$. or, $B = -1$

Thus $\frac{3x+1}{(x-1)(x+1)^2}$

$= \frac{1}{x-1} - \frac{1}{x+1} + \frac{1}{(x+1)^2}$

$\therefore \int \frac{x^4+3x^3+x^2-1}{x^3+x^2-x-1}\,dx$

$= \int (x+2)\,dx + \int \frac{dx}{x-1} - \int \frac{dx}{x+1} + \int \frac{dx}{(x+1)^2}$

$= \frac{x^2}{2} + 2x + \ln|x-1| - \ln|x+1|$

$- \frac{1}{x+1} + C$

(ii) Let $\frac{5x}{(x^2-2x+2)(x+1)}$

$= \frac{A}{x+1} + \frac{Bx+C}{x^2-2x+2}$ ———— (2)

$\Rightarrow \frac{5x}{(x^2-2x+2)(x+1)} = \frac{A(x^2-2x+2) + (Bx+C)(x+1)}{(x+1)(x^2-2x+2)}$

$\Rightarrow 5x = A(x^2-2x+2) + (Bx+C) \times (x+1)$

Put $x = -1$; then $5 \times (-1) = A(1+2+2)$
$+ (B \times (-1) + C) \times (-1+1)$

$\Rightarrow -5 = 5A + 0$

$\Rightarrow 5A = -5$

$\Rightarrow A = -\frac{5}{5}$

$\Rightarrow \boxed{A = -1}$

put $x = 0$, then

$0 = A(0-0+2) + (0+C)(0+1)$

$\Rightarrow 0 = (-1)(2) + C \times 1$

$\Rightarrow C - 2 = 0$

$\Rightarrow C = 2$

put $x = 1$, then $5 = (-1)(1-2+2) + (B+2)2$

$\Rightarrow 5 = -1 + 2B + 4$

$\Rightarrow 5 + 1 - 4 = 2B$

$\Rightarrow 2B = 2$

$\Rightarrow B = \frac{\cancel{2}}{\cancel{2}} = 1$

$\Rightarrow B = 1$

$$\therefore \int \frac{5x}{(x^2-2x+2)(x+1)}\,dx = \int\left(\frac{-1}{x+1} + \frac{x+2}{x^2-2x+2}\right)dx$$

$$= -\int \frac{1}{x+1}\,dx + \int \frac{2(x+2)}{2(x^2-2x+2)}\,dx$$

$$= -\ln|x+1| + \frac{1}{2}\int \frac{2x+4}{x^2-2x+2}\,dx$$

$$= -\ln|x+1| + \frac{1}{2}\int \frac{2x-2+4+2}{x^2-2x+2}\,dx$$

$$= -\ln|x+1| + \frac{1}{2}\int \frac{2x-2}{x^2-2x+2}\,dx + \frac{6}{2}\int \frac{dx}{x^2-3x+2}$$

$$= -\ln|x+1| + \frac{1}{2}\ln|x^2-2x+2| + 3\int \frac{dx}{(x-1)^2+(1)^2}$$

$$= -\ln|x+1| + \frac{1}{2}\ln|x^2-2x+2| + 3\tan^{-1}|x-1| + c$$

$$\left(\because \int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}\right)$$

(iii) $\int \frac{3}{x^3-1}\,dx$

Now $\frac{3}{x^3-1} = \frac{3}{x^3-1^3}$

$$= \frac{3}{(x-1)(x^2+x+1)}$$

$$= \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1} \quad \text{——(1)}$$

$$\therefore \frac{3}{(x-1)(x^2+x+1)} = \frac{A(x^2+x+1)+(Bx+C)(x-1)}{(x-1)(x^2+x+1)}$$

$$\Rightarrow 3 = A(x^2+x+1) + (Bx+C)(x-1)$$

Let $x=1$, then $3 = A(1+1+1) + (B\times 1+C)(1-1)$

$$3 = A(3) + (B+C)(0)$$

$\Rightarrow 3A = 3$

$\Rightarrow A = \frac{3}{3} = 1$

put $x = 0$, then

$3 = A(0+0+1) + (0+C)(-1)$

$\Rightarrow 3 = A - C$

$\Rightarrow 3 = 1 - C$

$\Rightarrow C = 1 - 3$

$\Rightarrow C = -2$

put $x = 2$, then

$3 = A(x^2+x+1) + (Bx+C)(x-1)$

$3 = A((2)^2+2+1) + (B\times 2 + (-2))$

$= A(4+2+1) + (2B-2) \times 1$

$= 1(7) + (2\times B) - 2$

$= 7 - 2 + 2B$

$\Rightarrow 3 = 5 + 2B$

$\Rightarrow 2B = 3 - 5$

$\Rightarrow 2B = -2$

$\Rightarrow B = \frac{-2}{2}$

$\boxed{B = -1}$

$\therefore \int \frac{3}{x^3-1}\,dx$

$$= \int \frac{1}{x-1}\,dx + \int \frac{-x-2}{x^2+x+1}\,dx$$

$$= \ln|x-1| - \frac{1}{2}\int \frac{2(x+2)}{x^2+x+1}\,dx$$

$$= \ln|x-1| - \frac{1}{2}\int \frac{2x+4}{x^2+x+1}\,dx$$

$$= \ln|x-1| - \frac{1}{2}\int \frac{2x+1+3}{x^2+x+1}\,dx$$

$$= \ln|x-1| - \frac{1}{2}\left[\int \frac{2x+1}{x^2+x+1}\,dx + \int \frac{3}{x^2+x+1}\,dx\right]$$

$$= \ln|x-1| - \frac{1}{2}\ln|x^2+x+1| - \frac{3}{2}\int \frac{dx}{x^2+x+1}$$

$$= \ln|x-1| - \frac{1}{2}\ln|x^2+x+1| - \frac{3}{2}\int \frac{dx}{(x)^2 + 2\cdot x\cdot\frac{1}{2} + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1}$$

$$= \ln|x-1| - \frac{1}{2}\ln|x^2+x+1| - \frac{3}{2}\int \frac{dx}{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \ln|x-1| - \frac{1}{2}\ln|x^2+x+1| - \frac{\cancel{3}}{\cancel{2}}\cdot\sqrt{3}\,\frac{\cancel{2}}{\cancel{\sqrt{3}}}\tan^{-1}\left(\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + C$$

$$\ln|x-1| - \frac{1}{2}\ln|x^2+x+1| - \sqrt{3}\tan^{-1}\left(\frac{2\left(x+\frac{1}{2}\right)}{\sqrt{3}}\right) + C$$

(iv) $\displaystyle\int \frac{x^5+x^4+x^3+x^2+4x+1}{x^3+1}\,dx$

$$\frac{x^5+x^4+x^3+x^2+4x+1}{x^3+1}$$

$$= (x^2+x+1) + \frac{3x}{x^3+1} \text{ —— (2)}$$

$x^3+1 \,)\, x^5+x^4+x^3+x^2+4x+1 \,(\, x^2+x+1$

$\quad \underline{-(x^5+x^2)}$

$\quad x^4+x^3+4x+1$

$\quad \underline{-(x^4+x)}$

$\quad x^3+3x+1$

$\quad \underline{-(x^3+1)}$

$\quad 3x$

Now, $\dfrac{3x}{x^3+1^3} = \dfrac{3x}{(x+1)(x^2-x+1)}$

$$= \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1}$$

$$\Rightarrow \frac{3x}{(x+1)(x^2-x+1)} = \frac{A(x^2-x+1)+(Bx+C)(x+1)}{(x+1)(x^2-x+1)}$$

$$\Rightarrow 3x = A(x^2-x+1)+(Bx+C)(x+1) \text{ —— (1)}$$

put $x=-1$,

$\Rightarrow 3\times(-1) = A((-1)^2-(-1)+1) + (B\times(-1)+C)(-1+1)$

$\Rightarrow -3 = A(1+1+1)+0$

$\Rightarrow 3A = -3$

$\Rightarrow A = \dfrac{-\cancel{3}}{\cancel{3}} = -1$

$\Rightarrow A = -1$

put $x=0$, so eqn (1) becomes

$3\times 0 = A(0-0+1) + (B\times 0+C)$

$\Rightarrow 0 = -1\times 1 + (0+C)\times 1$

$\Rightarrow 0 = -1+C$

$\Rightarrow \boxed{C=1}$

put $x=1$, so eqn (1) becomes

$3\times(1) = (-1)(1-1+1) + (B+1)\times 2$

$\Rightarrow 3 = -1 + 2B + 2$

$\Rightarrow 2B+1 = 3$

$\Rightarrow 2B = 3-1 = 2$

$\Rightarrow B = \frac{2}{2} = 1$

$\boxed{B = 1}$

So,

$$\int \frac{x^5+x^4+x^3+x^2+4x+1}{x^3+1}\,dx$$

$$= \int (x^2+x+1)\,dx - \int \frac{1}{(x+1)}\,dx + \frac{1}{2}\int \frac{2(x+1)}{x^2-x+1}\,dx$$

$$= \frac{x^3}{3} + \frac{x^2}{2} + x - \ln|x+1| + \frac{1}{2}\int \frac{2x+2}{x^2-x+1}\,dx$$

$$= \frac{x^3}{3} + \frac{x^2}{2} + x - \ln|x+1| + \frac{1}{2}\int \frac{2x-1+3}{x^2-x+1}\,dx$$

$$= \frac{x^3}{3} + \frac{x^2}{2} + x - \ln|x+1| + \frac{1}{2}\int \frac{2x-1}{x^2-x+1}\,dx$$

$$+ \frac{3}{2}\int \frac{1}{\left(x-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}\,dx$$

$$= \frac{x^3}{3} + \frac{x^2}{2} + x - \ln|x+1| + \frac{1}{2}\ln$$

$$|x^2-x+1| + \cancel{\frac{3}{2}} \times \frac{\sqrt{3}\,\cancel{2}}{\cancel{3}} \tan^{-1}\left(\frac{x-\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + c$$

$$= \frac{x^2}{3} + \frac{x^2}{2} + x - \ln|x+1| + \frac{1}{2}\ln|x^2-x+1|$$

$$+ \sqrt{3}\tan^{-1}\left|\frac{\frac{2x-1}{\cancel{2}}}{\frac{\sqrt{3}}{\cancel{2}}}\right| + c$$

$$= \frac{x^2}{3} + \frac{x^2}{2} + x - \ln|x+1| + \frac{1}{2}\ln$$

$$|x^2-x+1| + \sqrt{3}\tan^{-1}\left|\frac{2x-1}{\sqrt{3}}\right| + c$$

5(iii) $$\int \frac{x-5}{7-x^2+6x}\,dx = -\int \frac{x-5}{x^2-6x-7}\,dx$$

$$= -\frac{1}{2}\int \frac{(2x-6)-4}{x^2-6x-7}\,dx$$

$$= -\frac{1}{2}\int \frac{(2x-6)\,dx}{x^2-6x-7} + 2\int \frac{dx}{x^2-6x-7}$$

$= -\frac{1}{2}\ln|x^2-6x-7| + 2\int \frac{dx}{(x-3)^2-4^2}$

$= -\frac{1}{2}\ln|x^2-6x-7|$

$\quad + \frac{2}{2\cdot 4}\ln\left|\frac{x-3-4}{x-3+4}\right| + c$

$= -\frac{1}{2}\ln|x^2-6x-7|$

$\quad + \frac{1}{2}\ln\left|\frac{x-7}{x+1}\right| + c$

(c) for $\frac{x^2}{(x^2+3)(x^2+2)}$ let $x^2 = z$

$\therefore \frac{x^2}{(x^2+3)(x^2+2)} = \frac{z}{(z+3)(z+2)}$

$= \frac{A}{z+3} + \frac{B}{z+2}$ (say)

$= \frac{A(z+2) + B(z+3)}{(z+3)(z+2)}$

Now $z = A(z+2) + B(z+3)$ is an identity

putting $z = (-2)$ we have $B = -2$

putting $z = -3$ we have $A = 3$.

$\therefore \quad I = \int \frac{x^2\,dx}{(x^2+3)(x^2+2)}$

$$= 3\int \frac{dx}{x^2+3} - 2\int \frac{dx}{x^2+2}$$

$$= 3 \cdot \frac{1}{\sqrt{3}} \tan^{-1}\frac{x}{\sqrt{3}} - 2 \cdot \frac{1}{\sqrt{2}} \tan^{-1}\frac{x}{\sqrt{2}}$$

$$= \sqrt{3}\tan^{-1}\frac{x}{\sqrt{3}} - \sqrt{2}\tan^{-1}\frac{x}{\sqrt{2}}$$

(vi) Let $I = \int \frac{x^3\,dx}{x^4+3x^2+2}$

Putting $x^2 = t \Rightarrow 2x\,dx = dt$ we have

$$I = \frac{1}{2}\int \frac{t\,dt}{t^2+3t+2}$$

$$= \frac{1}{2}\int \frac{t\,dt}{(t+2)(t+1)}$$

Let $\frac{t}{(t+2)(t+1)} = \frac{A}{t+2} + \frac{B}{t+1}$

$$= \frac{A(t+1) + B(t+2)}{(t+2)(t+1)}$$

$\Rightarrow t = A(t+1) + B(t+2)$ is an identity.

Putting $t = (-1)$ we have

$B = -1$

Putting $t = -2$ we have

$-2 = -A \Rightarrow A = 2$

$\therefore\ I = \frac{1}{2}\left[2\int \frac{dt}{t+2} - \int \frac{dt}{t+1}\right]$

$= \frac{1}{2}\{2\log|t+2| - \log|t+1|\} + C$

$= \log|x^2+2| - \frac{1}{2}\log|x^2+1| + C$

(vii) Let $I = \int \frac{dx}{\sin x(3+2\cos x)} = \int \frac{\sin x\, dx}{\sin^2 x\,(3+2\cos x)}$

$= \int \frac{\sin x\, dx}{(1-\cos^2 x)(3+2\cos x)}$

Let $\cos x = t \Rightarrow -\sin x\, dx = dt$

$\therefore\ I = -\int \frac{dt}{(1+t)(1-t)(3+2t)}$

Let $\frac{-1}{(1-t)(1+t)(3+2t)}$

$= \frac{A}{1-t} + \frac{B}{1+t} + \frac{C}{3+2t}$

$= \frac{A(1+t)(3+2t) + B(1+t)(3+2t) + C(1+t)(1+t)}{(1-t)(1+t)(3+2t)}$

$\therefore\ -1 = A(1+t)(3+2t) + B(1-t)(3+2t) + C(1-t)(1+t)$ is an identity

Putting $t = 1$ we have

$1 = A(2)(5) \Rightarrow A = \frac{1}{10}$

Putting $t = (-1)$

$1 = B(2) \Rightarrow B = \frac{1}{2}$

Putting $t = \frac{3}{2}$

$1 = C\left(\frac{19}{4}\right) \Rightarrow C = \frac{4}{5}$

$\therefore I = \frac{1}{10}\int \frac{dt}{1-t} - \frac{1}{2}\int \frac{dt}{1+t}$

$+ \frac{4}{5}\int \frac{dt}{3+2t} + c$

$= \frac{1}{10}\log|1-t| - \frac{1}{2}\log|1+t|$

$+ \frac{4}{5}\cdot\frac{1}{2}\log|3+2t| + c$

$= -\frac{1}{2}\log(1+\cos x) + \frac{1}{10}\log(1-\cos x)$

$+ \frac{1}{2}\log(3+2\cos x) + c$