Exercise - 9(k)

1/(i) $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\tan x}$

$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

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$$\Rightarrow \int_0^{\frac{\pi}{2}} \frac{dx}{1+\tan x} = I$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{dx}{1+\tan\left(\frac{\pi}{2}-x\right)}$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{dx}{1+\cot x}$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{dx}{1+\frac{1}{\tan x}}$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{dx}{\frac{\tan x+1}{\tan x}}$$

$$= \int_0^{\frac{\pi}{2}} \frac{\tan x}{\tan x+1}\,dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\tan x+1-1}{\tan x+1}\,dx$$

$$I = \int_0^{\frac{\pi}{2}} \left[\left(\frac{\tan x+1}{\tan x+1}\right) - \frac{1}{\tan x+1}\right] dx$$

$$= \int_0^{\frac{\pi}{2}} 1\,dx - \int_0^{\frac{\pi}{2}} \frac{dx}{1+\tan x}$$

$$\Rightarrow I = x\Big]_0^{\frac{\pi}{2}} - I$$

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$$\Rightarrow I + I = x\Big]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{2} \times \frac{1}{2} = \frac{\pi}{4}$$

(ii) $$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx = I$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx \quad \text{—— (1)}$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin\left(\frac{\pi}{2} - x\right)}}{\sqrt{\sin\left(\frac{\pi}{2} - x\right)} + \sqrt{\cos\left(\frac{\pi}{2} - x\right)}} \quad \text{—— (2)}$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}}\, dx \quad \text{—— (2)}$$

Adding eqn (1) and (2)

$$\Rightarrow I + I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx + \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1\, dx = x\Big]_0^{\frac{\pi}{2}}$$

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$\Rightarrow 2I = \frac{\pi}{2}$

$\Rightarrow I = \frac{\pi}{2} \times \frac{1}{2} = \frac{\pi}{4}$

$\therefore \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx = \frac{\pi}{4}$

(iii) $\int_0^1 \frac{\ln(1+x)}{1+x^2} dx = I$

let $x = \tan\theta$

$\Rightarrow \theta = \tan^{-1}x$

$x = \tan\theta$

$\Rightarrow \frac{dx}{dx} = \frac{d\tan\theta}{d\theta} \cdot \frac{d\theta}{dx}$

$\Rightarrow 1 = \sec^2\theta \cdot \frac{d\theta}{dx}$

$\therefore dx = \sec^2\theta \, d\theta$

for $x = 0, \theta = 0$

for $x = 1, \theta = \tan^{-1}1$

$= \tan^{-1}\tan\frac{\pi}{4}$

$= \frac{\pi}{4}$

$\int_0^{\frac{\pi}{4}} \frac{\ln(1+\tan\theta)}{1+\tan^2\theta} \sec^2\theta \, d\theta$

$= \int_0^{\frac{\pi}{4}} \frac{\ln(1+\tan\theta)}{\sec^2\theta} \sec^2\theta \, d\theta$

$\left[\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx \right.$

$\Rightarrow I = \int_0^{\frac{\pi}{4}} \ln\left(1+\tan\left(\frac{\pi}{4}-\theta\right)\right) d\theta$

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$$= \int_0^{\frac{\pi}{4}} \ln\left(1+ \frac{\tan\frac{\pi}{4} - \tan\theta}{1+\tan\frac{\pi}{4}\cdot\tan\theta}\right) d\theta$$

$$= \int_0^{\frac{\pi}{4}} \ln\left(\frac{1+\cancel{\tan\theta}+1-\cancel{\tan\theta}}{1+\tan\theta}\right) d\theta$$

$$= \int_0^{\frac{\pi}{4}} \ln \frac{2}{1+\tan\theta}\, d\theta$$

$$= \int_0^{\frac{\pi}{4}} \left[\ln 2 - \ln(1+\tan\theta)\right] d\theta \qquad \left[\because \tan\frac{\pi}{4} = 1\right.$$

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$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \ln 2\, d\theta - \int_0^{\frac{\pi}{2}} \ln(1+\tan\theta)\, d\theta$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \ln 2\, d\theta - I$$

$$\Rightarrow I + I = \int_0^{\frac{\pi}{4}} \ln 2\, d\theta$$

$$= \ln 2\,\theta \Big]_0^{\frac{\pi}{4}}$$

$$\Rightarrow 2I = \ln 2 \times \frac{\pi}{4} - \ln 2 \times 0$$

$$\Rightarrow I = \frac{1}{2} \times \frac{\pi}{4} \ln 2$$

$$\Rightarrow \frac{\pi}{8} \ln 2$$

$$\therefore \int_0^1 \frac{\ln(1+x)}{1+x^2}\,dx = \frac{\pi}{8}\ln 2$$

(iv)

$$\int_0^{\pi} \frac{x}{1+\sin x}\,dx$$

$$I = \int_0^{\pi} \frac{x}{1+\sin x}\,dx \qquad \left[\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$$

$$\Rightarrow I = \int_0^{\pi} \frac{\pi - x}{1+\sin(\pi - x)}\,dx$$

$$= \int_0^{\pi} \frac{\pi}{1+\sin x}\,dx - \int_0^{\pi} \frac{x}{1+\sin x}\,dx$$

$$\Rightarrow I = \pi \int_0^{\pi} \frac{1}{1+\sin x}\,dx - I$$

$$\Rightarrow I + I = \pi \int_0^{\pi} \frac{dx}{1+\sin x}$$

$$\Rightarrow 2I = \pi \int_0^{\pi} \frac{1-\sin x}{(1+\sin x)(1-\sin x)}$$

$$= \pi \int_0^{\pi} \frac{(1-\sin x)}{(1-\sin^2 x)}\,dx$$

$$= \pi \int_0^{\pi} \frac{(1-\sin x)}{\cos^2 x}\,dx$$

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$$= \pi \left[\int_0^{\pi} \frac{dx}{\cos^2 x} - \int_0^{\pi} \frac{\sin x}{\cos^2 x}\, dx \right]$$

$$= \pi \left[\int_0^{\pi} \sec^2 x\, dx - \int_0^{\pi} \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}\, dx \right]$$

$$= \pi \left[(\tan x)_0^{\pi} - \int_0^{\pi} \sec x \tan x\, dx \right]$$

$$= \pi \left[\tan x \right]_0^{\pi} - \left[\sec x\right]_0^{\pi}$$

$$= \pi \left[\tan\pi - \tan 0 - \sec\pi - \sec 0 \right]$$

$$= \pi \left[0 - (-1-1) \right]$$

$$\Rightarrow 2I = \pi \cdot 2$$

$$\Rightarrow I = \frac{2\pi}{2} = \pi$$

$$\therefore \int_0^{\pi} \frac{x}{1+\sin x}\, dx = \pi$$

Q/(1) $\int_{-a}^{a} x^2\, dx$

Here x^2 is even function

$$= 2\int_0^{a} x^2\, dx$$

[If $\int_{-a}^{a} f(x)\, dx = \begin{cases} 2\int_0^{a} f(x)\, dx, & \text{if } f \text{ is even} \\ 0, & \text{if } f \text{ is odd} \end{cases}$]

($f(-x) = f(x)$ even

$f(-x) = -f(x)$ odd)

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$= 2 \left. \frac{x^5}{5} \right]_0^a$

$= 2\left(\frac{a^5}{5} - 0\right)$

$= \frac{2}{5} a^5$

(or)

(i) $\int_{-a}^{a} x^4 \, dx$

$= \left. \frac{x^5}{5} \right]_{-a}^{a}$

$= \frac{1}{5}\left(a^5 - (-a)^5\right)$

$= \frac{1}{5}\left(a^5 + a^5\right)$

$= \frac{2}{5} a^5$

(iv) $\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \sin^5 x \, dx$

$\sin^5(-x) = (-\sin x)^5$

$= -\sin^5 x$

odd function

Here $\sin^5 x$ is odd funn,

$\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \sin^5 x \, dx = 0$

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2/(ii) $\int_{-a}^{a}(x^5+2x^2+x)\,dx = \left[\frac{x^6}{6}+\frac{2x^3}{3}+\frac{2x^2}{2}\right]_{-a}^{a}$

$= \frac{1}{6}(a^6-a^6)+\frac{2}{3}(a^3+a^3)+(a^2-a^2) = \frac{4a^3}{3}$

(iii) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos^2 x\,dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left[\frac{1+\cos x}{2}\right] dx$

$= \frac{1}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} dx + \frac{1}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos 2x\,dx$

$= \frac{1}{2}\left[x\right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}} + \frac{1}{4}\left[\sin 2x\right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$

$= \frac{1}{2}\left(\frac{\pi}{4}+\frac{\pi}{4}\right) + \frac{1}{4}\left(\sin\frac{\pi}{2}+\sin\frac{\pi}{2}\right) = \frac{\pi}{4}+\frac{1}{2}$

3/(i) $\int_{0}^{\pi} \cos^3 x\,dx = \int_{0}^{\pi} \frac{\cos 3x + 3\cos x}{4}\,dx$

$= \frac{1}{4}\left[\int_{0}^{\pi} \cos 3x\,dx + 3\int_{0}^{\pi} \cos x\,dx\right]$

$= \frac{1}{4}\left\{\left[\frac{1}{3}\sin 3x\right]_{0}^{\pi} + 3\left[\sin x\right]_{0}^{\pi}\right\}$

$= \frac{1}{12}(\sin 3\pi - \sin 0) + \frac{3}{4}(\sin \pi - \sin 0) = 0$

(ii) $\int_{0}^{\pi} \cos^2 x\,dx = \int_{0}^{\pi} \frac{1+\cos 2x}{2}\,dx$

$= \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right]_0^{\pi} = \frac{1}{2}\cdot\pi = \frac{\pi}{2}$

(iii) $\int_0^{\pi} \sin^3 x \cdot \cos x \, dx$

put $\sin x = t$, then $\cos x \, dx = dt$

when $x = 0$, when $x = \pi$, $t = 0$

$= \int_0^{0} t^3 \, dt = 0$

(iv) $\int_0^{\pi} \cos^2 x \cdot \sin x \, dx$

put $\cos x = t$, Then $\sin x \, dx = -dt$

when $x = 0$, $t = 1$

when $x = \pi$, $t = -1$

$= \int_1^{-1} t^2(-dt) = \int_{-1}^{1} t^2 \, dt = \left(\frac{t^3}{3}\right)_{-1}^{1}$

$= \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

5/(i) $I = \int_0^1 \frac{\ln x}{\sqrt{1-x^2}} \, dx$

Let $x = \sin u \Rightarrow u = \sin^{-1} x$

$dx = \cos u \, du$

for $x = 0$, $u = 0$

$x = 1$, $u = \frac{\pi}{2}$

$I = \int_0^{\frac{\pi}{2}} \frac{\ln \sin u}{\sqrt{1-\sin^2 u}} \cdot \cos u \, du$

$= \int_0^{\frac{\pi}{2}} \frac{\ln \sin u}{\cos u} \cdot \cos u \, du$

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$$= \int_0^{\frac{\pi}{2}} \ln(\sin x)\,dx \quad \text{——— (1)}$$

we know, $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$$I = \int_0^{\frac{\pi}{2}} \ln \sin\left(\frac{\pi}{2} - x\right) dx$$

$$= \int_0^{\frac{\pi}{2}} \ln \cos x \, dx \quad \text{——— (2)}$$

Adding eqn (1) and (2) we get

Now $I + I = \int_0^{\frac{\pi}{2}} \ln(\sin x)\,dx + \int_0^{\frac{\pi}{2}} \ln(\cos x)\,dx$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \ln\left(\frac{2\sin x \cdot \cos x}{2}\right) dx \quad \left(\begin{matrix}2 \text{ ଗୁଣିଲେ} \\ 2 \text{ ହରିଲେ}\end{matrix}\right)$$

$$= \int_0^{\frac{\pi}{2}} \ln\left(\frac{\sin 2x}{2}\right) dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \ln(\sin 2x)\,dx - \int_0^{\frac{\pi}{2}} \ln 2 \, dx$$

Let $2x = z$

$2dx = dz$

$\Rightarrow dx = \frac{dz}{2}$

For $x = 0,\ z = 0$

$x = \frac{\pi}{2},\ z = \pi$

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$$2I = \int_0^{\pi} \ln(\sin z)\, dz - \ln 2\, u \Big]_0^{\frac{\pi}{2}}$$

$$= \int_0^{\pi} \ln \sin z\, dz - \ln \left(\frac{\pi}{2}\right)$$

$$2I = I - \ln 2\left(\frac{\pi}{2}\right)$$

$$2I - I = \frac{\pi}{2} \ln 2^{-1}$$

$$\Rightarrow \int_0^1 \frac{\ln x}{\sqrt{1-x^2}}\, dx = \frac{\pi}{2} \ln \frac{1}{2}$$

$$\int_0^{2a} f(x)\, dx = 2\int_0^{a} f(x)\, dx$$

$$f(2a-x) = f(x)$$

$f(2a-x) = f(x)$
ଯଦି = ସମାନେ
$(\pi - z) \to$ 1st

(iii) $$\int_0^{\pi} x \ln(\sin x)\, dx = \frac{\pi^2}{2} \ln \frac{1}{2}$$

Let $$I = \int_0^{\pi} (\pi - x) \ln(\sin(\pi - x))\, dx$$

$$\left(\because \int_0^{a} f(x)\, dx = \int_0^{a} f(a-x)\, dx\right)$$

$$= \int_0^{\pi} \pi \ln(\sin x)\, dx - \int_0^{\pi} x \ln(\sin x)\, dx$$

$$\Rightarrow I = \pi \int_0^{\pi} \ln(\sin x)\, dx - I$$

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$$\Rightarrow I + I = \pi \int_0^{\pi} \ln(\sin x)\, dx$$

$$\Rightarrow 2I = \pi \int_0^{\pi} \ln(\sin x)\, dx$$

Here $f(x) = \ln \sin x$

$$f(\pi - x) = \ln \sin(\pi - x)$$
$$= \ln(\sin x)$$
$$= f(x)$$

$$\Rightarrow 2I = 2\pi \int_0^{\frac{\pi}{2}} \ln(\sin x)\, dx \text{ —— (2)}$$

Let $p = \int_0^{\frac{\pi}{2}} \ln(\sin x)\, dx$

$$= \int_0^{\frac{\pi}{2}} \ln \sin\left(\frac{\pi}{2} - x\right) dx$$

$$= \int_0^{\frac{\pi}{2}} \ln(\cos x)\, dx$$

$$\Rightarrow p + p = \int_0^{\frac{\pi}{2}} \ln(\sin x)\, dx + \int_0^{\frac{\pi}{2}} \ln(\cos x)\, dx$$

$$\Rightarrow 2p = \int_0^{\frac{\pi}{2}} \ln\left(\frac{2 \sin x \cdot \cos x}{2}\right) dx$$

$$= \int_0^{\frac{\pi}{2}} \ln(\sin 2x)\, dx - \int_0^{\frac{\pi}{2}} \ln 2\, dx$$

$\Rightarrow 2p = p - \int_0^{\frac{\pi}{2}} \ln 2 \, dx$

$\Rightarrow p = -\ln 2 \, [x]_0^{\frac{\pi}{2}}$

$= \ln (2)^{-1} \left(\frac{\pi}{2} - 0\right)$

$= \frac{\pi}{2} \ln \frac{1}{2}$

Now eqn(1) becomes.

$\int_0^{\pi} x \ln(\sin x) \, dx = \frac{\pi^2}{2} \ln \frac{1}{2}$

5/(v) $\int_0^1 x(1-x)^{100} \, dx$ $\left[\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx \right]$

$= \int_0^1 (1-x)\left(1-(1-x)\right)^{100} dx$

$= \int_0^1 (1-x)(1-1+x)^{100} \, dx$

$= \int_0^1 (1-x)x^{100} \, dx$

$= \int_0^1 (x^{100} - x^{101}) \, dx$

$= \left[\frac{x^{101}}{101} - \frac{x^{102}}{102} \right]_0^1$

$= \frac{1}{101} - \frac{1}{102} = \frac{1}{10302}$

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(i) $I = \int_0^{\pi/2} \ln(\tan x + \cot x)\, dx$

$= \int_0^{\pi/2} \ln\left(\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}\right) dx$

$= \int_0^{\pi/2} \ln\left(\frac{1}{\cos x \sin x}\right) dx$

$= \int_0^{\pi/2} \ln\left\{(\cos x)^{-1}(\sin x)^{-1}\right\} dx$

$= \int_0^{\pi/2} \ln(\cos x)\, dx - \int_0^{\pi/2} \ln(\sin x)\, dx$

$= -2\int_0^{\pi/2} \ln(\sin x)\, dx$

$\left[\because \int_0^{\pi/2} \ln(\cos x)\, dx = \int_0^{\pi/2} \ln(\sin x)\, dx\right]$

$= -2\cdot\left(-\frac{\pi}{2}\ln 2\right) = \pi \ln 2$

(ii) $I = \int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$

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$$= \int_0^{\pi} \frac{(\pi-x)\tan(\pi-x)}{\sec(\pi-x)+\tan(\pi-x)}\,dx$$

$$= \int_0^{\pi} \frac{(\pi-x)\tan x}{\sec x+\tan x}\,dx$$

$$= \int_0^{\pi} \frac{\pi\tan x}{\sec x+\tan x}\,dx - \int_0^{\pi} \frac{x\tan x}{\sec x+\tan x}\,dx$$

$$= \pi\int_0^{\pi} \frac{(\sec x-\tan x)\tan x}{\sec^2 x-\tan^2 x}\,dx \quad 1$$

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$$\Rightarrow 2I = \pi\int_0^{\pi} (\sec x\tan x - \tan^2 x)\,dx$$

$$= \pi\int_0^{\pi} (\sec x\tan x - \sec^2 x + 1)\,dx$$

$$= \pi\left[\sec x - \tan x + x\right]_0^{\pi}$$

$$= \pi\left[(-1-0+\pi)-(1-0+0)\right]$$

$$= \pi(\pi-2)$$

$$\Rightarrow I = \pi\left(\frac{\pi}{2}-1\right)$$

(iii) $$I = \int_1^3 \frac{\sqrt{x}\,dx}{\sqrt{4-x}+\sqrt{x}}$$

$$= \int_1^3 \frac{\sqrt{1+3-x}}{\sqrt{4-(1+3-x)}+\sqrt{1+3-x}}\,dx$$

$$= \int_1^3 \frac{\sqrt{4-x}}{\sqrt{x}+\sqrt{4-x}}\,dx$$

$$\therefore 2I = I + I$$

$$= \int_1^3 \frac{\sqrt{x}}{\sqrt{4-x}+\sqrt{x}}\,dx + \int_1^3 \frac{\sqrt{4-x}}{\sqrt{x}+\sqrt{4-x}}\,dx$$

$$= \int_1^3 dx = [x]_1^3 = 3-1 = 2$$

$$\Rightarrow I = 1$$

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(iv) $I = \int_0^\pi \frac{x\sin x}{1+\cos^2 x}\,dx$

$$= \int_0^\pi \frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)}\,dx$$

$$= \pi\int_0^\pi \frac{\sin x}{1+\cos^2 x}\,dx - \int_0^\pi \frac{x\sin x}{1+\cos^2 x}\,dx$$

$$= \pi\int_0^\pi \frac{\sin x}{1+\cos^2 x}\,dx - I$$

$$\therefore 2I = \int_0^\pi \frac{\sin x}{1+\cos^2 x}\,dx$$

$$= \pi \int_1^{-1} \frac{dt}{1+t^2}$$

where $\cos x = t$, $-\sin x \, dx = dt$

$$= \pi \int_{1}^{-1} \frac{dt}{1+t^2}$$

$x = 0 \Rightarrow t = 1$

$x = \pi \Rightarrow t = -1$

$$= \pi \left[\tan^{-1} t\right]_{-1}^{1}$$

$$= \pi \left[\tan^{-1} 1 - \tan^{-1}(-1)\right]$$

$$= \pi \left[\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right] = \frac{\pi^2}{2}$$

$$\therefore \quad I = \frac{\pi^2}{4}$$

(vi) $$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\cot x}}$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}\, dx}{\sqrt{\sin x}+\sqrt{\cos x}}$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin\left(\frac{\pi}{6} - \frac{\pi}{3} - x\right)}}{\sqrt{\sin\left(\frac{\pi}{6}+\frac{\pi}{3}-x\right)}+\sqrt{\cos\left(\frac{\pi}{6}+\frac{\pi}{3}-x\right)}}\, dx$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\, dx$$

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$$2I = I + I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx \quad \left[\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx\right]$$

$$+ \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} dx = [x]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$$

$$\Rightarrow I = \frac{\pi}{12}$$

(viii) $$\int_0^{50} e^{x-[x]}\,dx \quad \left[\int_0^{n} f(x)\,dx = n\int_0^{T} f(x)\,dx,\ n \in N\right]$$

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$$f(x) = e^{x-[x]}$$

$$f(x+1) = e^{x+1-[x+1]}$$
$$= e^{x+1-\{[x]+1\}}$$
$$= e^{x+1-[x]-1}$$
$$= e^{x-[x]} = f(x)$$

The funⁿ is a periodic funⁿ

of period $T = 1$

$$\int_0^{50\times 1} e^{x-[x]}\,dx$$

$$=50\int_0^1 e^{x-[x]}\,dx$$

$$=50\int_0^1 e^{x-0}\,dx$$

$$=50\int_0^1 e^{x}\,dx=50\left(e^1-e^0\right)$$

$$=50(e-1)$$

Date - 25.12.23

4/(ii) $$I=\int_0^{\frac{\pi}{2}}\frac{\cos x-\sin x}{1+\sin x\cos x}\,dx$$

$$=\int_0^{\frac{\pi}{2}}\frac{\cos\left(\frac{\pi}{2}-x\right)-\sin\left(\frac{\pi}{2}-x\right)}{1+\sin\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-x\right)}\,dx$$

$$=\int_0^{\frac{\pi}{2}}\frac{\sin x-\cos x}{1+\sin x-\cos x}\,dx$$

$$\therefore 2I=\int_0^{\frac{\pi}{2}}\frac{\cos x-\sin x}{1+\sin x\cos x}\,dx+\int_0^{\frac{\pi}{2}}\frac{\sin x-\cos x}{1+\sin x\cos x}\,dx$$

$$=\int_0^{\frac{\pi}{2}}0\,dx=0$$

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