Date → 27.12.23

## Differential Equation

If an equation involves differential or derivative then it is called so. When the derivative is partial order then the equation is called partial differential equation like

$$\frac{\partial^2 u}{\partial x^2} - 2x\frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial y^2} = 0$$

When the eqn having ordinary derivative then it is called ordinary differential equation.

Ex: $\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 5y = 0$

Ex: $\frac{d^2y}{dx^2} = 3\left(\frac{dy}{dx}\right)^3 + 5x = 0$

Order and degree of the differential equation: order and degree of the equation depends on the highest derivative of the equation. Order of the highest derivative is the order of the eqn and the degree of highest derivative is the equation can not be fraction.

ex:- $\frac{d^2y}{dx^2} - 3\left(\frac{dy}{dx}\right)^4 + 5y = 0$

order of the eqn is 2

degree of the eqn is 1

ex:- $\frac{d^2y}{dx^2} = 4\left(\frac{dy}{dx}\right)^3 - x\frac{dy}{dx}$

order is 2

degree is 1

## formation of differential eqn :-

Taken derivative in both side of a linear eqn we get the differential eqn.

If any arbitary consists are there then we have elliminate them. Ent

$x + y = 5$

$\frac{dx}{dy} + \frac{dy}{dx} = \frac{d5}{dx}$

$1+\dfrac{dy}{dx}=0$

$\dfrac{dy}{dx}=-1$

Ex:-

$y=a\sin x$

$\dfrac{dy}{dx}=a\cos x$

$=\dfrac{y}{\sin x}\cdot\cos x=y\cot x$

$\dfrac{dy}{dx}=y\cot x$



Ex:-

$y=a\cos x+b\sin x$ —(1)

$y_1=a(-\sin x)+b\cos x$

$=-a\sin x+b\cos x$ —(2)

$y_2=-a\cos x-b\sin x$

$=-(a\cos x+b\sin x)$ —(ii)

$y_2=-y$

$y_2+y=0$

Soln of differential eqn

1st order differential eqn: In case of 1st order differential eqn we have to separate the differentials w.r.t their variables and integrate them soln is obtained.

Ex:-

$$\frac{dy}{dx} = xy$$

$$dy = xy\,dx$$

$$\frac{dy}{y} = x\,dx$$

$$= \int \frac{dy}{y} = \int x\,dx$$

$$\ln y = \frac{x^2}{2} + c$$

$$y = e^{\frac{x^2}{2} + c}$$

Ex:-

$$\frac{dy}{dx} = \frac{e^{2x}+1}{e^x}$$

$$dy = \frac{e^{2x}+1}{e^x}\,dx$$

$$\int dy = \int \frac{e^{2x}}{e^x}\,dx + \int \frac{1}{e^x}\,dx$$

$$\int dy = \int (e^x + e^{-x})\,dx$$

$$y = e^x - e^{-x} + c$$

• In case of 2nd order differential eqn 1st we have to expression it as 1st order by substitute 1st order as new variable. Then proceed as before.

Ex:

$$\frac{d^2y}{dx^2} = x^2$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = x^2 \quad \left[\text{let } \frac{dy}{dx} = t\right.$$

$$\frac{dt}{dx} = x^2$$

$$dt = x^2 dx$$

$$\int dt = \int x^2 dx$$

$$t = \frac{x^3}{3} + c_1$$

$$\frac{dy}{dx} = \frac{x^3}{3} + c_1$$

$$dy = \frac{x^3}{3} dx + c_1 dx$$

$$\int dy = \int \frac{x^3}{3} dx + \int c_1 dx$$

$$y = \frac{1}{3} \cdot \frac{x^4}{4} + c_1 x + c_2$$

Exercise - 11 (a)

1/ (i) $y \sec^2 x \, dx + \tan x \, dy = 0$

Ans → order = 1 , Degree = 1

ii/ $\left(\frac{dy}{dx}\right)^4 + y^5 = \frac{d^3y}{dx^3}$

Ans → order = 3 degree = 1

iii/ $a \frac{d^2y}{dx^2} = \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}$

Ans → order = 2 degree = 2

iv/ $\tan^{-1}\sqrt{\frac{dy}{dx}} = x$

Ans → order = 1 , degree = 1

v/ $\ln\left(\frac{d^2y}{dx^2}\right) = y$

Ans → order = 2 degree = 1

vi/ $\frac{\frac{dy}{dt}}{y + \frac{dy}{dt}} = \frac{yt}{\frac{dy}{dt}}$

Ans → order = 1 degree = 2

(vii) $\frac{d^2y}{dx^2} = \frac{3y + \frac{dy}{dx}}{\sqrt{\frac{d^2y}{dx^2}}}$

Ans → Order = 2 , degree = 3

viii) $\dfrac{dy}{e^{dx}} = x^2$

Ans → order = 1, degree = 1

2) From the differential equation by eliminating the arbitary constants in each of the following cases.

(i) $y = A\sec x$

Then $\dfrac{dy}{dx} = A\sec x\tan x = y\tan x$

(ii) $y = c\tan^{-1}x \Rightarrow \dfrac{y}{\tan^{-1}x} = c$

$\Rightarrow \dfrac{d}{dx}\left(\dfrac{y}{\tan^{-1}x}\right) = 0$

$\Rightarrow \dfrac{\dfrac{dy}{dx}\cdot\tan^{-1}x - y\cdot\dfrac{1}{1+x^2}}{(\tan^{-1}x)^2} = 0$

$\Rightarrow \tan^{-1}x\dfrac{dy}{dx} - \dfrac{y}{1+x^2} = 0$

(iii) $y = Ae^{t} + Be^{2t}$ ——— (1)

$\dfrac{dy}{dt} = Ae^{t} + 2Be^{2t}$ ——— (2)

subtracting (1) from (2) we get

$$\frac{dy}{dt} - y = Be^{2t}$$

$$\Rightarrow \left(\frac{dy}{dt} - y\right) \cdot e^{-2t} = B$$

Differentiating again we get

$$\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right) \cdot e^{-2t} + \left(\frac{dy}{dt} - y\right)\left(-2e^{-2t}\right) = 0$$

$$\Rightarrow \frac{d^2y}{dt^2} - \frac{dy}{dt} - 2\frac{dy}{dt} + 2y = 0$$

$$\Rightarrow \frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 0$$

(iv)

$$y = Ax^2 + Bx \Rightarrow y = (Ax+B)x$$

$$\Rightarrow \frac{y}{x} = Ax + B$$

$$\Rightarrow \frac{xy_1 - y}{x^2} = A$$ ← Differentiating both side

$$\Rightarrow \frac{x^2(xy_2 + y_1 - y_1) - (xy_1 - y)\cdot 2x}{x^4} = 0$$ Again differentiating

$$\Rightarrow x^2y_2 - 2xy_1 + 2y = 0$$

(v)

$$y = a\cos x + b\sin x$$

$$\frac{dy}{dx} = -a\sin x + b\cos x$$

$$\frac{d^2y}{dx^2} = -a\cos x - b\sin x = -y$$

$$= -(a\cos x + b\sin x) = -y$$

$$\Rightarrow \frac{d^2y}{dx^2} + y = 0$$

(vi)

$$y = a\sin^{-1}x + b\cos^{-1}x$$

$$\Rightarrow \frac{dy}{dx} = \frac{a}{\sqrt{1-x^2}} - \frac{b}{\sqrt{1-x^2}}$$

$$\Rightarrow \sqrt{1-x^2}\,\frac{dy}{dx} = a-b$$

$$\Rightarrow \frac{d}{dx}\left(\sqrt{1-x^2}\right)\frac{dy}{dx} + \sqrt{1-x^2}\cdot\frac{d^2y}{dx^2} = 0$$

$$\Rightarrow \frac{-2x}{2\sqrt{1-x^2}}\,\frac{dy}{dx} + \sqrt{1-x^2}\,\frac{d^2y}{dx^2} = 0$$

$$\Rightarrow \sqrt{1-x^2}\,\frac{d^2y}{dx^2} + \frac{x}{\sqrt{1-x^2}}\cdot\frac{dy}{dx} = 0$$

$$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$$



(vii) $y = at + be^t$

$$\Rightarrow \frac{dy}{dt} = a + be^t$$

$$\Rightarrow \frac{d^2y}{dt^2} = be^t$$

putting the value of '$be^t$' in eqn (2)

~~By (3) we get~~ from (2))

$$\frac{dy}{dt} = a + \frac{d^2y}{dt^2} \Rightarrow a = \frac{dy}{dt} - \frac{d^2y}{dt^2}$$

putting the value of a and $be^t$ in (1) we get

$$y = t\left(\frac{dy}{dt} - \frac{d^2y}{dt^2}\right) + \frac{d^2y}{dt^2}$$

$$\Rightarrow (1-t)\frac{d^2y}{dt^2} + t\frac{dy}{dt} - y = 0$$

(viii)

$y = a\sin t + be^t$

$\frac{dy}{dt} = a\cos t + be^t$

$\frac{d^2y}{dt^2} = -a\sin t + be^t$

Adding (1) and (3) we get

$$y + \frac{d^2y}{dt^2} = 2be^t \Rightarrow be^t = \frac{y + \frac{d^2y}{dt^2}}{2}$$

Again from (1) we get

$$a\sin t = y - be^t = y - \frac{y + \frac{d^2y}{dt^2}}{2} = \frac{y - \frac{d^2y}{dt^2}}{2}$$

from (2) we get

$$a\cos t = \frac{dy}{dt} - \frac{y + \frac{d^2y}{dt^2}}{2} = \frac{2\frac{dy}{dt} - y - \frac{d^2y}{dt^2}}{2}$$

$$\therefore \frac{a\cos t}{a\sin t} = \frac{2\frac{dy}{dt} - y - \frac{d^2y}{dt^2}}{y - \frac{d^2y}{dt^2}}$$

$$\Rightarrow (\cot t - 1)\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - (\cot t + 1)y = 0$$

(ix) $ax^2 + by = 1$ $\left[y = -\frac{a}{b}x^2 + \frac{1}{b}\right]$

$\Rightarrow by = 1 - ax^2 \Rightarrow y = \frac{1}{b} - \frac{ax^2}{b}$

$\Rightarrow \frac{dy}{dx} = -\frac{2a}{b}x \Rightarrow \frac{\frac{dy}{dx}}{x} = -\frac{2a}{b}$

Again differentiating we get

$$\frac{\frac{d^2y}{dx^2}\cdot x - \frac{dy}{dx}}{x^2} = 0 \Rightarrow \frac{d^2y}{dx^2}x - \frac{dy}{dx} = 0$$

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3/(i) $\frac{dy}{dx} = \frac{e^{2x+1}}{e^x} \Rightarrow dy = \frac{e^{2x+1}}{e^x} dx$

$\Rightarrow \int dy = \int \left(\frac{e^{2x}}{e^x}\right) dx$

$\Rightarrow y = \int (e^x + e^{-x})\, dx = e^x - e^{-x} + c$

(ii) $\frac{dy}{dx} = x\cos x \Rightarrow y = \int x \cdot \cos x\, dx$

$= x \cdot \sin x - \int \sin x\, dx = x\sin x + \cos x + c$

(iii) $\frac{dy}{dt} = t^5 \cdot \log t \Rightarrow dy = \int t^5 \cdot \log t\, dt$

$\Rightarrow y = \int \log t \cdot t^5\, dt$

$= \log t \cdot \frac{t^6}{6} - \int \frac{1}{t} \cdot \frac{t^6}{6} dt$

$= \frac{1}{6} t^6 \log t - \frac{1}{6} \int t^5\, dt$

$= \frac{1}{6} t^6 \log t - \frac{1}{36} t^6 + c$

(iv) $\frac{dy}{dt} = 3t^2 + 4t + \sec^2 t$

$\Rightarrow y = t^3 + 2t^2 + \tan t + c$

(v) $\frac{dy}{dx} = \frac{1}{x^2 - 7x + 12}$

$\Rightarrow y = \int \frac{dx}{x^2 - 7x + 12} = \int \frac{dx}{(x-3)(x-4)}$

$$= \int \left\{ \frac{1}{x-4} - \frac{1}{x-3} \right\} dx = \ln \frac{x-4}{x-3} + c$$

(vi) $$\frac{dy}{dx} = \frac{x+1}{\sqrt{3x^2+6x+5}}$$

$$\Rightarrow y = \int \frac{x+1}{\sqrt{3x^2+6x+5}} dx$$

$$= \frac{1}{6} \int \frac{6x+6}{\sqrt{3x^2+6x+5}} dx$$

$$= \frac{1}{6} \frac{(3x^2+6x+5)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c$$

$$= \frac{1}{3} \sqrt{3x^2+6x+5} + c$$



(vii) $$(x^2+3x+2)\,dy - dx = 0$$

$$\Rightarrow dy = \frac{dx}{x^2+3x+2}$$

$$\Rightarrow y = \int \frac{dx}{(x+1)(x+2)}$$

$$= \int \left( \frac{1}{x+1} - \frac{1}{x+2} \right) dx = \ln\left(\frac{x+1}{x+2}\right) + c$$

(viii) $$\frac{dy}{dt} = \frac{\sin^{-1} t \cdot e^{\sin^{-1} t}}{\sqrt{1-t^2}}$$

$$\Rightarrow y = \int \frac{\sin^{-1} t \cdot e^{\sin^{-1} t}}{\sqrt{1-t^2}}$$

put $\sin^{-1} t = u$, then $\frac{dt}{\sqrt{1-t^2}} = du$

$$= \int u \cdot e^u du = u \cdot e^u - e^u + c$$

$$= \sin^{-1} t \cdot e^{\sin^{-1} t} - e^{\sin^{-1} t} + c$$

4/(i) $\dfrac{dy}{dx} = y+2$

$\left[ \therefore e^c = c \right.$

$\dfrac{dy}{dx} = y+2 \Rightarrow \int \dfrac{dy}{y+2} = \int dx$

$\Rightarrow \ln(y+2) = \int x + c_1$

$\Rightarrow y+2 = e^{x+c_1} = e^x \cdot e^{c_1} = ce^x$

where $c = e^{c_1}$

(ii) $\dfrac{dy}{dt} = \sqrt{1-y^2} \Rightarrow \int \dfrac{dy}{\sqrt{1-y^2}} = dt$

$\Rightarrow \sin^{-1} y = t+c$

(iii) $\dfrac{dy}{dz} = \sec y \Rightarrow \int \cos y \, dy = \int dz$

$\Rightarrow \sin y = z+c$

(iv) $\dfrac{dy}{dx} = e^y \Rightarrow e^{-y} dy = dx$

Integrating we get $\int e^{-y} dy = \int dx$

$\Rightarrow \dfrac{e^{-y}}{-1} = x+c \Rightarrow -e^{-y} + x + c$

$\Rightarrow x + e^{-y} + c = 0$

(v) $\dfrac{dy}{dx} = y^2 + 2y \Rightarrow \dfrac{dy}{y(y+2)} = dx$

$\Rightarrow \int \left(\dfrac{1}{y} - \dfrac{1}{y+2}\right) \dfrac{1}{2} dy = \int dy$

$\Rightarrow \frac{1}{2}\ln\frac{y}{y+2} = x + c$

(vi) $dy + (y^2+1)dx = 0$

$\Rightarrow \frac{dy}{1+y^2} + dx = 0$

$\Rightarrow \int \frac{dy}{1+y^2} + \int dx = c$

$\Rightarrow \tan^{-1} y + x = c$

(vii) $\frac{dy}{dx} + \frac{e^y}{y} = 0$

$\Rightarrow y e^{-y} dy + dx = 0$

$\Rightarrow \int y e^{-y} dy + \int dx = c$

$\Rightarrow y \cdot \frac{e^{-y}}{-1} - \int 1 \cdot \frac{e^{-y}}{-1} dy + x = c$

$\Rightarrow -ye^{-y} + \frac{e^{-y}}{-1} + x = c$

$\Rightarrow -ye^{-y} - e^{-y} + x = c$

$\Rightarrow x - (y+1)e^{-y} = c$

(viii) $dx + \cot x \, dt = 0$

$\Rightarrow \tan x \, dx + dt = 0$

$\Rightarrow \int \tan x \, dx + \int dt = c_1$

$\Rightarrow \ln \sec x + t = c_1$

$\Rightarrow \ln \sec x = c_1 - t$

$\Rightarrow \sec x = e^{c_1} \cdot e^{-t}$

$\Rightarrow \cos x = e^{-c_1} \cdot e^t$

$\Rightarrow \cos x = ce^t$ where $c = e^{-c_1}$

5/ (i) $\frac{dy}{dx} = (x^2+1)(y^2+1)$

$\Rightarrow \frac{dy}{(1+y^2)} = (x^2+1)\,dx$

$\Rightarrow \int\frac{dy}{1+y^2} = \int(x^2+1)\,dx$

$\Rightarrow \tan^{-1} y = \frac{x^3}{3} + x + c$



(ii) $\frac{dy}{dt} = e^{2t+3y} = e^{2t} \cdot e^{3y}$

$\Rightarrow e^{-3y}\,dy = e^{2t}\,dt$

$\Rightarrow \int e^{-3y} \cdot dy = \int e^{2t}\,dt$

$\Rightarrow \frac{e^{-3y}}{-3} = \frac{e^{2t}}{2} + c_1$

$\Rightarrow \frac{1}{3}e^{-3y} + \frac{1}{2}e^{2t} + c_1 = 0$

$\Rightarrow 2e^{-3y} + 3e^{2t} + 6c_1 = 0$

$\Rightarrow 2e^{-3y} + 3e^{2t} = c$

where $c = -6c_1$

(iii) $\frac{dy}{dz} = \frac{\sqrt{1-y^2}}{\sqrt{1-z^2}}$

$$\Rightarrow \frac{dy}{\sqrt{1-y^2}} = \frac{dz}{\sqrt{1-z^2}}$$

$$\Rightarrow \int \frac{dy}{\sqrt{1-y^2}} = \int \frac{dz}{\sqrt{1-z^2}}$$

$$\Rightarrow \sin^{-1} y = \sin^{-1} z + C$$

(iv) $$\frac{dy}{dx} = \frac{x \ln x}{3y^2+4y}$$

$$\Rightarrow (3y^2+4y)\,dy = \int x \ln x \cdot dx$$

$$\Rightarrow \int (3y^2+4y)\,dy = \int x \cdot \ln x \, dx$$

$$\Rightarrow y^3 + 2y^2 = \frac{x^2}{2} \cdot \ln x - \int \frac{1}{x} \cdot \frac{x^2}{2}\,dx$$

$$= \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C$$

(v) $$x^2\sqrt{y^2+3}\,dx + y\sqrt{x^3+1}\,dy = 0$$

$$\Rightarrow \frac{x^2}{\sqrt{x^3+1}}\,dx + \frac{y}{\sqrt{y^2+3}}\,dy = 0$$

$$\Rightarrow \frac{1}{3}(x^3+1)^{-\frac{1}{2}} \cdot 3x^2\,dx + \frac{1}{2}(y^2+3)^{-\frac{1}{2}} \cdot 2y\,dy = 0$$

$$\Rightarrow \frac{1}{3}\int (x^3+1)^{-\frac{1}{2}} \cdot 3x^2\,dx + \frac{1}{2}(y^2+3)^{-\frac{1}{2}} \cdot 2y\,dy = c$$

$$\Rightarrow \frac{\frac{1}{3}(x^3+1)^{\frac{1}{2}}}{\frac{1}{2}} + \frac{1}{2}\,\frac{(y^2+3)^{\frac{1}{2}}}{\frac{1}{2}} = c$$

$\Rightarrow \dfrac{dy}{\sqrt{1-y^2}} = \dfrac{dz}{\sqrt{1-z^2}}$

$\Rightarrow \int \dfrac{dy}{\sqrt{1-y^2}} = \int \dfrac{dz}{\sqrt{1-z^2}}$

$\Rightarrow \sin^{-1} y = \sin^{-1} z + C$

(iv) $\dfrac{dy}{dx} = \dfrac{x \ln x}{3y^2+4y}$

$\Rightarrow (3y^2+4y)\,dy = x \ln x \cdot dx$

$\Rightarrow \int (3y^2+4y)\,dy = \int x \cdot \ln x\, dx$

$\Rightarrow y^3 + 2y^2 = \dfrac{x^2}{2} \cdot \ln x - \int \dfrac{1}{x} \cdot \dfrac{x^2}{2}\, dx$

$= \dfrac{1}{2} x^2 \ln x - \dfrac{1}{4} x^2 + C$

(v) $x^2 \sqrt{y^2+3}\, dx + y\sqrt{x^3+1}\, dy = 0$

$\Rightarrow \dfrac{x^2}{\sqrt{x^3+1}} dx + \dfrac{y}{\sqrt{y^2+3}} dy = 0$

$\Rightarrow \dfrac{1}{3} (x^3+1)^{-\frac{1}{2}} \cdot 3x^2 dx + \dfrac{1}{2} (y^2+3)^{-\frac{1}{2}} \cdot 2y\, dy = 0$

$\Rightarrow \dfrac{1}{3} \int (x^3+1)^{-\frac{1}{2}} \cdot 3x^2 dx + \dfrac{1}{2} \int (y^2+3)^{-\frac{1}{2}} \cdot 2y\, dy = c$

$\Rightarrow \dfrac{\frac{1}{3}(x^3+1)^{\frac{1}{2}}}{\frac{1}{2}} + \dfrac{1}{2} \dfrac{(y^2+3)^{\frac{1}{2}}}{\frac{1}{2}} = c$

$$\left[\because \int [f(x)]^n \cdot f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1}\right.$$

$$\Rightarrow \frac{2}{3}\sqrt{x^3+1} + \sqrt{y^2+3} = c_1$$

$$\Rightarrow 2\sqrt{x^3+1} + 3\sqrt{y^2+3} = 3c_1$$

$$\Rightarrow 2\sqrt{x^3+1} + 3\sqrt{y^2+3} = c$$

where $c = 3c_1$

(vi) $\tan y\,dx + \cot x\,dy = 0$

$$\Rightarrow \tan x \cdot dx + \cot y\,dy = 0$$

$$\Rightarrow \int \tan x\,dx + \int \cot y\,dy = 0$$

$$\Rightarrow -\ln\cos x + \ln\sin y = \ln c$$

$$\Rightarrow \ln\frac{\sin y}{\cos x} = \ln c$$

$$\Rightarrow \frac{\sin y}{\cos x} = c \Rightarrow \sin y = c\cos x$$

(vii) $(x^2+7x+12)\,dy + (y^2-6y+5)\,dx = 0$

$$\Rightarrow \frac{dy}{y^2-6y+5} + \frac{dx}{x^2+7x+12} = 0$$

$$\Rightarrow \frac{dy}{(y-1)(y-5)} + \frac{dx}{(x+3)(x+4)} = \frac{1}{4}\ln c$$

$$\Rightarrow 4\ln\frac{x+3}{x+4} + \ln\frac{y-5}{y-1} = \ln c$$

$$\Rightarrow \int\left(\frac{1}{x+3}-\frac{1}{x-4}\right) \cdot dx +$$

$$\int\left(\frac{1}{y-5}-\frac{1}{y-1}\right) \cdot \frac{1}{4} = \frac{1}{4}\ln c$$

$$\Rightarrow \ln\frac{x+3}{x+4}+\frac{1}{4}\ln\frac{y-5}{y-1} \ln c$$

$$\Rightarrow \ln\left\{\left(\frac{x+3}{x+4}\right)^4 \cdot \frac{y-5}{y-1}\right\} = \ln c$$

$$\Rightarrow \frac{(x+3)^4 (y-5)}{(x+4)^4 (y-1)} = c$$

$$\Rightarrow (x+3)^4 (y-5) = c\,(x+4)^4 (y-1)$$

(viii) $y\,dy + e^{-y}\sin x\,dx = 0$

$$\Rightarrow y\,e^{y}\,dy + x\sin x\,dx = 0$$

$$\Rightarrow \int y e^{y}\,dy + \int x\sin x\,dx = c$$

(Integrating by parts)

$$\Rightarrow ye^{y} - \int e^{y}\,dy + x\cdot(-\cos x) - \int 1\cdot(-\cos x)\,dx = c$$

$$\Rightarrow ye^{y} - e^{y} - x\cos x + \sin x = c$$

$$\Rightarrow (y-1)\,e^{y} - x\cos x + \sin x = c$$

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(i) $\frac{d^2y}{dx^2} = 12x^2 + 2x$

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = 12x^2 + 2x$ $\Big[$ Let $\frac{dy}{dx} = t$

$\frac{dt}{dx} = 12x^2 + 2x.$

$dt = (12x^2 + 2x)\,dx$

$\int dt = \int 12x^2\,dx + \int 2x\,dx$

$t = 12\cdot\frac{x^3}{3} + 2\cdot\frac{x^2}{2} + c_1$

$= 4x^3 + x^2 + c_1$

$\frac{dy}{dx} = 4x^3 + x^2 + c_1$

$dy = 4x^3\,dx + x^2\,dx + c_1\,dx$

$\int dy = \int 4x^3\,dx + \int x^2\,dx \int c_1\,dx$

$y = 4\cdot\frac{x^4}{4} + \frac{x^3}{3} + c_1x + c_2$

$y = x^4 + \frac{x^3}{3} + c_1x + c_2$

(ii) $\frac{d^2y}{dt^2} = e^{2t} + e^{-t}$ Let $\frac{dy}{dt} = z$

$\frac{d}{dt}\left(\frac{dy}{dt}\right) = e^{2t} + e^{-t}$

$$dz = (e^{2t} + e^{-t})\,dt$$

$$\int dz = \int e^{2t}\,dt + \int e^{-t}\,dt$$

$$z = \frac{1}{2}e^{2t} - e^{-t} + c_1$$

$$\frac{dy}{dt} = \frac{1}{2}e^{2t} - e^{-t} + c_1$$

$$dy = \frac{1}{2}e^{2t}\,dt - e^{-t}\,dt + c_1\,dt$$

$$\int dy = \int \frac{1}{2}e^{2t}\,dt - \int e^{-t}\,dt + \int c_1\,dt$$

$$y = \frac{1}{2}\cdot\frac{1}{2}e^{2t} + e^{-t} + c_1 t + c$$

$$y = \frac{1}{4}e^{2t} + e^{-t} + c_1 t + c_2$$



(iii) $$\frac{d^2y}{du^2} = -\sin u + \cos u + \sin^2 u$$

$$\frac{d}{du}\left(\frac{dy}{du}\right) = -\sin u + \cos u + \sec^2 u$$

$$\frac{dz}{du} = -\sin u + \cos u + \sec^2 u$$

$$dz = -\sin u\,du + \cos u\,du + \sec^2 u\,du$$

$$\int dz = \int -\sin u\,du + \int \cos u\,du + \int \sec^2 u\,du$$

$$z = \cos u + \sin u + \tan u + c_1$$

$$\frac{dy}{du} = \cos u + \sin u + \tan u + c_1$$

$$\int dy = \int \cos u\,du + \int \sin u\,du + \int \tan u\,du + \int c_1\,du$$

$y = \sin v - \cos v + \ln|\sec v| + c_1 v + c_2$

(iv) $\operatorname{cosec} x \dfrac{d^2y}{dx^2} = x \Rightarrow \dfrac{d^2y}{dx^2} = x\sin x$

Integrating we get

$\dfrac{dy}{dx} = \int x\sin x\,dx + A$

$= x\cdot(-\cos x) - \int 1\cdot(-\cos x)\,dx + A$

$= -x\cos x + \int \cos x\,dx + A$

$= -x\cos x + \sin x + A$

Again integrating we get

$y = -\int x\cos x\,dx + \int \sin x\,dx$
$\quad + \int A\,dx + B$

$= -\{x\cdot\sin x - \int 1\cdot\sin x\,dx\} - \cos x + Ax + B$

$= -x\sin x - 2\cos x + Ax + B$

(v) $x^2 \dfrac{d^2y}{dx^2} + 2 = 0$

$\Rightarrow \dfrac{d^2y}{dx^2} = \dfrac{-2}{x^2}$

$\Rightarrow \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{-2}{x^2}$ $\left[\text{Let } \frac{dy}{dx} = z\right.$

$\Rightarrow \frac{dz}{dx} = \frac{-2}{x^2}$

$\Rightarrow dz = \frac{-2}{x^2}dx$

$\Rightarrow \int dz = \int \frac{-2}{x^2}dx \Rightarrow z = -2\int x^{-2}dx$

$\Rightarrow z = -2 \cdot \frac{x^{-1}}{-1} \Rightarrow z = 2\frac{1}{x} + c_1$

$\frac{dy}{dx} = \frac{2}{x} + c_1 \Rightarrow dy = \frac{2}{x}dx + c_1 dx$

$\Rightarrow dy = 2\frac{dx}{x} + c_1 dx$

$\int dy = \int 2\frac{dx}{x} + \int c_1 dx \Rightarrow y = 2\ln x + c_1 x + c_2$

(vi) $\sec x \frac{d^2y}{dx^2} = \sin 3x$ $\left[\sin(A+B) + \sin(A-B) = 2\sin A \cdot \cos B\right.$

$\Rightarrow \frac{d^2y}{dx^2} = \sin 3x \cdot \cos x$

$= \frac{1}{2}(2\sin 3x \cdot \cos x)$

$= \frac{1}{2}(\sin 4x + \sin 2x)$

Integrating we get

$\frac{dy}{dx} = \frac{1}{2}\left\{\int \sin 4x\,dx + \int \sin 2x\,dx\right\} + A$

$= \frac{1}{2} \cdot \frac{-\cos 4x}{4} + \frac{1}{2}\cos 2x + A$

Again integrating we get.

$y = -\frac{1}{32}\sin 4x - \frac{1}{8}\sin 2x + Ax + B$

(vii) $\cancel{\frac{d^2y}{dx^2} = \sec}$

(viii) $\frac{d^2y}{dx^2} = \sec^2 x + \cos^2 x$ [Let $\frac{dy}{dx} = z$

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \sec^2 x + \cos^2 x$

$\frac{dz}{dx} = \sec^2 x + \cos^2 x$

$dz = \sec^2 x\,dx + \cos^2 x\,dx$

$\int dz = \int \sec^2 x\,dx + \int \cos^2 x\,dx$

$z = \tan x + \int \frac{1+\cos 2x}{2}\,dx$

$= \tan x + \int \frac{1}{2}\,dx + \int \frac{\cos 2x}{2}\,dx$

$= \tan x + \frac{1}{2}x + \frac{1}{2}\cdot\frac{1}{2}\sin 2x + c_1$

$= \tan x + \frac{1}{2}x + \frac{1}{4}\sin 2x + c_1$

$\frac{dy}{dx} = \tan x + \frac{1}{2}x + \frac{1}{4}\sin 2x + c_1$

$\int dy = \int \tan x\,dx + \int \frac{1}{2}x\,dx + \int \frac{1}{4}\sin 2x\,dx + \int c_1\,dx$

$y = \ln|\sec x| + \frac{1}{2}\frac{x^2}{2} - \frac{1}{4}\cdot\frac{1}{2}\cos 2x + c_1x + c_2$

$= \log|\sec x| + \frac{1}{8}\left(\sin 2x - \cos 2x\right) + c_1 x + c_2$

(viii)

$e^{-x} \frac{d^2y}{dx^2} = x$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{x}{e^{-x}}$

$\Rightarrow \frac{d}{dx}\left(\frac{dy}{dx}\right) = e^x \cdot x$ (Let $\frac{dy}{dx} = z$)

$\Rightarrow \frac{dz}{dx} = e^x x$

$\Rightarrow dz = e^x x\, dx$

$\Rightarrow \int dz = \int e^x x\, dx$

$\Rightarrow z = \left(\int e^x dx\right) x - \int\left(\int e^x dx\right)\frac{dx}{dx}\, dx$

$= xe^x - \int e^x dx$

$= xe^x - e^x + c_1$

$\frac{dy}{dx} = xe^x - e^x + c_1$

$\int dy = \int xe^x dx - \int e^x dx + \int c_1 dx$

$= \cancel{xe^x} = \int(e^x dx)\, x - \int\left(\int e^x dx\right)\frac{dx}{dx} dx - \int e^x dx + c_1 x$

$= xe^x - \int e^x dx - e^x + c_1 x$

$= xe^x - e^x - e^x + c_1 x + c_2$

$= xe^x - 2e^x + c_1 x + c_2$

7(i) $\frac{dy}{dx} = \cos x$, $y = 2$, when $x = 0$

$\int dy = \int \cos x \, dx$

$y = \sin x + c$

$x = 0,\ y = 2 \Rightarrow 2 = \sin 0 + c$

$\Rightarrow 2 = 0 + c \Rightarrow c = 2$

Required particular soln

$y = \sin x + 2$

(ii) $\frac{dy}{dt} = \cos^2 y$, $y = \frac{\pi}{4}$, $t = 0$



$\frac{dy}{\cos^2 y} = dt$

$\int \sec^2 y \, dy = \int dt$

$\tan y = t + c$

$c = \tan \frac{\pi}{4} - 0 = 1 - 0 = 1$

Required particular soln.

$\tan y = t + 1$

(iii) $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ $y = \sqrt{3}$, $x = 1$

$\Rightarrow dy\,(1+x^2) = dx\,(1+y)^2$

$\Rightarrow \frac{dy}{(1+y^2)} = \int \frac{dx}{(1+x^2)}$

$\Rightarrow \tan^{-1} y = \tan^{-1} x + c$

$y = \sqrt{3}$, $x = 1 \Rightarrow \tan^{-1}\sqrt{3} = \tan^{-1} 1 + c$

$\Rightarrow c = \tan^{-1}\sqrt{3} - \tan^{-1} 1$

$= \tan^{-1}\tan\frac{\pi}{3} - \tan^{-1}\tan\frac{\pi}{4}$

$= \frac{\pi}{3} - \frac{\pi}{4}$

$= \frac{4\pi - 3\pi}{12} = \frac{\pi}{12}$

Required particular soln.

$\tan^{-1} y = \tan^{-1} x + \frac{\pi}{12}$

(iv) $\frac{d^2y}{dx^2} = 6x$, $y = 1$, $\frac{dy}{dx} = 2$, $x = 0$

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = 6x$

$\frac{dt}{dx} = 6x$

$\int dt = \int 6x\,dx$

$t = 6 \cdot \frac{x^2}{2} + c_1$

$\frac{dy}{dx} = 3x^2 + c_1$

$\int dy = \int 3x^2\,dx + \int c_1\,dx$

$y = 3 \cdot \frac{x^3}{3} + c_1 x + c_2$

$x = 0$, $\frac{dy}{dx} = 2 \Rightarrow 2 = 3 \cdot 0 + c_1$

$x = 0$, $y = 1 \Rightarrow 1 = 3 \cdot 0 + 2 \cdot 0 + c_2$

$\Rightarrow c_2 = 1$

Required particular soln

$= x^3 + 2x + 1$

8/(i) $\frac{dy}{dx} = \sec(x+y)$

Let $x+y = t \Rightarrow 1 + \frac{dy}{dx} = \frac{dt}{dx}$

$\therefore \frac{dt}{dx} - 1 = \sec t$

$\Rightarrow \frac{dt}{dx} = 1 + \sec t$

$\Rightarrow \int \frac{dt}{1+\sec t} = \int dx$

$\Rightarrow \int \left(1 - \frac{1}{1+\cos t}\right) dt = \int dx$

$\Rightarrow \int \left(1 - \frac{1}{2\cos^2 \frac{t}{2}}\right) dt = \int dx$

$\Rightarrow \int \left(1 - \frac{1}{2}\sec^2 \frac{t}{2}\right) dt = \int dx$

$\Rightarrow t - \tan \frac{t}{2} = x + c$

$\Rightarrow x+y - \tan\left(\frac{x+y}{2}\right) = x + c$

$\Rightarrow y - \tan\left(\frac{x+y}{2}\right) = c$

(ii) $\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$

Let $x+y = t \Rightarrow 1 + \frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{dt}{dx} = 1 + \sin t + \cos t$

$\Rightarrow \int \frac{dt}{1 + \sin t + \cos t} = \int dx$

$\Rightarrow \int \frac{1+\tan^2 \frac{t}{2}\, dt}{1 + \tan^2 \frac{t}{2} + \tan \frac{t}{2} + 1 - \tan^2 \frac{t}{2}} = \int dx$

$$\Rightarrow \int \frac{\sec^2\frac{t}{2}\,dt}{2\left(1+\tan\frac{t}{2}\right)} = dx$$

$$\Rightarrow \int \frac{du}{u} = \int dx$$

where $1+\tan\frac{t}{2} = u$

$\frac{1}{2}\sec^2\frac{t}{2} = du$

$\Rightarrow \ln|u| = x+c$

$\Rightarrow \ln\left|1+\tan\frac{x+y}{2}\right| = x+c$

(iii) $\frac{dy}{dx} = \cos(x+y)$

Let $x+y = t \Rightarrow 1+\frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{dt}{dx} - 1 = \cos t$

$$\Rightarrow \int \frac{dt}{1+\cos t} = \int dx$$

$$\Rightarrow \int \frac{dt}{2\cos^2\frac{t}{2}} = \int dx$$

$$\Rightarrow \frac{1}{2}\int \sec^2\frac{t}{2}\,dt = \int dx$$

$\Rightarrow \tan\frac{t}{2} = x+c$

$\Rightarrow \tan\left(\frac{x+y}{2}\right) = x+c$

(iv) $\frac{dy}{dx} + 1 = e^{x+y}$

Let $x+y = t \Rightarrow 1+\frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{dt}{dx} = e^{t}$

$\Rightarrow \int e^{-t}\, dt = \int dx$

$-e^{-t} = x + c \Rightarrow \dfrac{-e^{-t}}{e^{-t}} = \dfrac{x+c}{e^{-t}} = e^{t}(x+c)$

$\Rightarrow -1 = e^{t}(x+c)$

$\Rightarrow e^{x+y}(x+c) + 1 = 0$