Date : 06.01.23

Homogenous Differential equation : A differential equation of the form $\frac{dy}{dx} = \frac{f(x, y)}{g(x, y)}$ is said to be homogenous if both $f(x,y)$ and $g(x,y)$ are of same degree in every terms. To separate the differential w.r.t their variables we have ~~too~~ to substitute $y = vx$ in both ~~sa~~ side of the equation then integrating both side soln will ~~obta~~ obtain.

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Ex :-

$$(x^2+y^2)\,dx - 2xy\,dy = 0$$

$$(x^2+y^2)\,dx = 2xy\,dy$$

$$\frac{dy}{dx} = \frac{x^2+y^2}{2xy} \text{ homogeneous}$$

Let $y = vx$

$$\frac{dy}{dx} = v + x\cdot\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{x^2+v^2x^2}{2xvx}$$

$$= \frac{\cancel{x^2}(1+v^2)}{\cancel{2x^2v}} = \frac{(1+v^2)}{2v}$$

$$x\frac{dv}{dx} = \frac{(1+v^2)}{2v} - v$$

$$= \frac{1+v^2-2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{1-v^2}{2v}$$

$$\frac{dv}{\frac{1-v^2}{2v}} = \frac{dx}{x}$$

$$\int\frac{2v\,dv}{1-v^2} = -\int\frac{dx}{x}$$

$$2\int\frac{v\,dv}{1-v^2} = -\ln x + \ln c$$

Let $1-v^2 = t$

$-2v = \frac{dt}{dv}$

$2v\,dv = -\frac{dt}{2}$

$$2\int\frac{dt}{2t} = -\ln x + \ln c$$

$$\frac{2}{2}\int\frac{dt}{t} = -\ln x + c$$

$$\ln t = -\ln x + c$$

$$\ln(1-v^2) = -\ln x + c$$

$$\ln(1-v^2) = -\ln x + c$$

$$\ln\left(1-\frac{y^2}{x^2}\right) = \ln\frac{c}{x}$$

$$\frac{x^2-y^2}{x^2} = \frac{c}{x}$$

$$x^2-y^2 = cx$$

$$y^2 = x^2 - cx$$

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Exercise – 19 (c)

(2) $(x+y)\,dy + (x-y)\,dx = 0$

Ans → Given equation can be written as

$$\frac{dy}{dx} = \frac{x-y}{x+y}$$

This is

$$(x+y)\,dy = -(x-y)\,dx$$

$$\frac{dy}{dx} = \frac{-(x-y)}{x+y}, \text{ homogeneous}$$

Let $y = vx$

$$\frac{dy}{dx} = v + \frac{dv}{dx}\cdot x$$

$$= v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{-(x-y)}{x+y}$$

$$x\cdot\frac{dv}{dx} = \frac{-x+vx}{x+vx} - v$$

$$= \frac{-x+vx - vx - v^2x}{x+vx}$$

$$= \frac{-v^2x - x}{x+vx}$$

$$= \frac{x(v^2+v)}{x(1+v)}$$

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$$\int \frac{(1+v)\,dv}{(v^2+1)} = -\int \frac{dx}{x}$$

Let $v^2+1 = t$

$2v = \frac{dt}{dv}$

$v\,dv = \frac{dt}{2}$

$$\int \frac{dv}{v^2+1} + \int \frac{v\,dv}{v^2+1} = -\ln x + c$$

$$\tan^{-1} v + \frac{1}{2}\int \frac{dt}{t} = -\ln x + c$$

$$\tan^{-1} v + \frac{1}{2}\ln(v^2+1) = -\ln x + c$$

$$\tan^{-1} v + \ln\sqrt{v^2+1} = -\ln x + c$$

$$\tan^{-1} v = -\ln x - \ln\sqrt{v^2+1} + c$$

$$= -\left\{\ln x + \ln\sqrt{v^2+1}\right\} + c$$

$$-\tan^{-1}\frac{y}{x} = \ln \cancel{x}\sqrt{\frac{y^2+x^2}{\cancel{x^2}}} + c$$

$$\tan^{-1}\frac{y}{x} = \ln\sqrt{y^2+x^2} + c$$

$$\ln\sqrt{y^2+x^2} + \tan^{-1}\frac{y}{x} + c = 0$$

2/ $$\frac{dy}{dx} = \frac{1}{2}\left(\frac{y}{x} + \frac{y^2}{x^2}\right)$$

$$= \frac{1}{2}\left(\frac{xy+y^2}{x^2}\right), \text{homogenous}$$

Let $y = vx$

$$v + x\frac{dv}{dx} = \frac{dy}{dx}$$

$$v + x\frac{dv}{dx} = \frac{1}{2}\left(\frac{xy+y^2}{x^2}\right)$$

$$x\frac{dv}{dx} = \frac{1}{2}\left\{\frac{xy+y^2}{x^2}\right\} - v$$

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$$= \frac{xy + y^2}{2x^2} - v$$

$$= \frac{x vx + v^2 x^2}{2x^2} - v$$

$$= \frac{vx^2 + v^2 x^2 - 2vx^2}{2x^2}$$

$$= \frac{\cancel{x^2}(v + v^2 - 2v)}{2\cancel{x^2}}$$

$$x\frac{dv}{dx} = \frac{v^2 - v}{2}$$

$$\frac{dv}{\frac{v^2 - v}{2}} = \frac{dx}{x}$$

$$\int \frac{dv}{v^2 - v} = \frac{1}{2}\int \frac{dx}{x}$$

$$\int \frac{dv}{v^2 - v} = \frac{1}{2}\ln x + \ln c$$

$$\int \frac{dv}{v(v-1)} = \frac{1}{2}\ln x + \ln c$$

$$\int \left(\frac{1}{v-1} - \frac{1}{v}\right) dv = \frac{1}{2}\ln x + \ln c$$

$$\int \frac{dv}{v-1} - \int \frac{dv}{v} = \ln \sqrt{x} + \ln c$$

$$\ln \frac{(v-1)}{v} = \ln c\sqrt{x}$$

$$\ln \frac{\frac{y}{x} - 1}{\frac{y}{x}} = \ln c\sqrt{x} \Rightarrow \frac{y-x}{y} = c\sqrt{x}$$

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$y - x = cy\sqrt{x}$

(3) $(x^2 - y^2)\,dx + 2xy\,dy = 0$

$(x^2 - y^2)\,dx = -2xy\,dy$

$-2xy\,dy = (x^2 - y^2)\,dx$

$\frac{dy}{dx} = -\frac{(x^2 - y^2)}{2xy}$, homogeneous

Let $y = vx$

$v + x\frac{dv}{dx} = \frac{dy}{dx}$

$v + x\frac{dv}{dx} = -\frac{(x^2 - y^2)}{2xy}$

$x\frac{dv}{dx} = -\frac{(x^2 - v^2x^2)}{2xvx} - v$

$= \frac{-x^2 + v^2x^2 - 2v^2x^2}{2x^2v}$

$= \frac{\cancel{x^2}(-1 - v^2)}{2\cancel{x^2}v}$

$= \frac{(-1 - v^2)}{2v}$

$\frac{dv}{\frac{(-1 - v^2)}{2v}} = \frac{dx}{x}$

$\int \frac{2v\,dv}{1 + v^2} = -\int \frac{dx}{x}$

Let $1 + v^2 = t$

$2v = \frac{dt}{dv}$

$2v\,dv = dt$

$\int \frac{dt}{t} = -\ln x + \ln c$

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$\ln t = -\ln x + \ln c$

$\ln(1+v^2) = -\ln x + \ln c$

$\ln\left(1+\dfrac{y^2}{x^2}\right) = -\ln x + \ln c$

$\ln \dfrac{x^2+y^2}{x^2} + \ln x = \ln c$

$\ln \dfrac{x^2+y^2}{\cancel{x}^2} \cdot \cancel{x} = \ln c$

$\ln \dfrac{(x^2+y^2)}{x} = \ln c$

$\dfrac{x^2+y^2}{x} = c$

$x^2+y^2 = cx$

(4) $x\dfrac{dy}{dx} + \sqrt{x^2+y^2} = y$

$x\dfrac{dy}{dx} = y - \sqrt{x^2+y^2}$

$\dfrac{dy}{dx} = \dfrac{y - \sqrt{x^2+y^2}}{x}$

Let $y = vx$

$\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

$v + x\dfrac{dv}{dx} = \dfrac{vx - \sqrt{x^2+v^2x^2}}{x}$

$x\dfrac{dv}{dx} = \dfrac{vx - \sqrt{x^2(1+x^2)}}{x} - v$

$= \dfrac{\cancel{vx} - x\sqrt{1+x^2} - \cancel{vx}}{x}$

$x\dfrac{dv}{dx} = \dfrac{-\cancel{x}\sqrt{1+v^2}}{\cancel{x}}$

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$$\int \frac{dv}{\sqrt{1+v^2}} = -\int \frac{dx}{x}$$

$$\ln\left(v+\sqrt{v^2+1}\right) = -\ln x + \ln c$$

$$\ln\left(\frac{y}{x}+\sqrt{\frac{y^2}{x^2}+1}\right) = \ln\frac{c}{x}$$

$$\ln\left(\frac{y}{x}+\sqrt{\frac{y^2+x^2}{x^2}}\right) = \ln\frac{c}{x}$$

$$\frac{y+\sqrt{y^2+x^2}}{x} = \frac{c}{x}$$

$$\frac{x}{\sqrt{y^2+x^2}} = c-y$$

(5) $x(x+y)\,dy = (x^2+y^2)\,dx$

$$\frac{dy}{dx} = \frac{x^2+y^2}{x(x+y)}$$

Let $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{x^2+y^2}{x(x+y)}$$

$$x\frac{dv}{dx} = \frac{x^2+v^2x^2}{x^2+x\cdot vx} - v$$

$$x\frac{dv}{dx} = \frac{x^2+v^2x^2}{x^2+vx^2} - v$$

$$= \frac{x^2+\cancel{v^2x^2}-vx^2-\cancel{v^2x^2}}{x^2+vx^2}$$

$$= \frac{x^2-vx^2}{x^2+vx^2}$$

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$$x\frac{dv}{dx} = \frac{\cancel{x^2}(1-v)}{\cancel{x^2}(1+v)}$$

$$\frac{\frac{dv}{1-v}}{1+v} = \frac{dx}{x}$$

$$\int\frac{(1+v)\,dv}{1-v} = \int\frac{dx}{x}$$

$$\int\frac{dv}{1-v} + \int\frac{v\,dv}{1-v} = \ln x + C$$

Let $1-v=t$
$-1 = \frac{dt}{dv}$
$dv = -dt$

$$\int\frac{dv}{1-v} - \int\frac{1-v-1}{1-v}\,dv = \ln x + C$$

$$\ln(1-v) - \int dv + \int\frac{dv}{1-v} = \ln x + C$$

$$\ln(1-v) - v + \ln(1-v) = \ln x + C$$

$$\ln(1-v)^2 - v = \ln x + C$$

$$\ln\left(1-\frac{y}{x}\right)^2 - \frac{y}{x} = \ln x + C$$

$$\ln\frac{(x-y)^2}{x^2} - \frac{y}{x} = \ln x + C$$

$$-\frac{y}{x} = \ln\frac{x}{\frac{(x-y)^2}{x^2}} + C$$

$$\frac{y}{x} = \ln\frac{x}{(x-y)^2} + C$$

$$e^{\frac{y}{x}} = \frac{x}{c} \quad x = e^{\frac{y}{x}}\,c\,(x-y)^2$$

(e)

$$y^2 + x^2\frac{dy}{dx} = xy\frac{dy}{dx}$$

$$x^2\frac{dy}{dx} - xy\frac{dy}{dx} = -y^2$$

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$$(x^2 - xy)\frac{dy}{dx} = -y^2$$

$$\frac{dy}{dx} = \frac{-y^2}{x^2 - xy}$$

let $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{-v^2x^2}{x^2 - x^2v} = \frac{-v^2x^2}{x^2(1-v)}$$

$$x\frac{dv}{dx} = \frac{-v^2\cancel{x^2}}{\cancel{x^2}(1-v)} - v$$

$$x\frac{dv}{dx} = \frac{-\cancel{v^2} - v + \cancel{v^2}}{1-v}$$

$$\frac{(1-v)}{v}dv = -\frac{dx}{x}$$

$$\int\frac{dv}{v} - \int\frac{v\,dv}{v} = -\ln x + c$$

$$\ln v - v = \ln \frac{c}{\cancel{x}}$$

$$-v = \ln cx - \ln v + c$$

$$v - \frac{y}{x} = \ln\frac{x}{\frac{y}{x}} + c$$

$$\frac{y}{x} = -\ln\frac{1}{y} + c$$

$$\frac{y}{x} = \ln y + c$$

$$-\frac{x}{y} = \ln\frac{cy}{x^2}$$

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7/ $x \sin\frac{y}{x}\, dy = \left(y \sin\frac{y}{x} - x\right) dx$

$$\frac{dy}{dx} = \frac{y \sin\frac{y}{x} - x}{x \sin\frac{y}{x}}$$

Let $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{y \sin\frac{y}{x} - x}{x \sin\frac{y}{x}}$$

$$x\frac{dv}{dx} = \frac{y \sin\frac{y}{x} - x}{x \sin\frac{y}{x}} - v$$

$$= \frac{vx \sin\frac{vx}{x} - x}{x \sin\frac{vx}{x}} - v$$

$$= \frac{vx \sin v - x - vx \sin x}{x \sin v}$$

$$x\frac{dv}{dx} = \frac{-\cancel{x}}{\cancel{x} \sin v}$$

$$x\frac{dv}{dx} = -\operatorname{cosec} v$$

$$\int \frac{dv}{-\operatorname{cosec} v} = \int \frac{dx}{x}$$

$$\int -\sin v\, dv = \ln x + c$$

$$\cos v = \ln x + c$$

$$\cos\left(\frac{y}{x}\right) = \ln x + c$$

$$\ln x = \cos\left(\frac{y}{x}\right) + c$$

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8/ $x\,dy - y\,dx = \sqrt{x^2+y^2}\,dx$

$x\,dy = \sqrt{x^2+y^2}\,dx + y\,dx$

$x\,dy = \left(\sqrt{x^2+y^2} + y\right)dx$

$\frac{dy}{dx} = \frac{\left(y + \sqrt{x^2+y^2}\right)}{x}$

Let $y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{y + \sqrt{x^2+y^2}}{x}$

$x\frac{dv}{dx} = \frac{y + \sqrt{x^2+y^2}}{x} - v$

$= \frac{vx + \sqrt{x^2+v^2x^2}}{x} - v$

$= \frac{\cancel{vx} + \sqrt{x^2(1+v^2)}}{x} - \cancel{vx}$

$x\frac{dv}{dx} = \frac{\cancel{x}\sqrt{1+x^2}}{\cancel{x}}$

$\int \frac{dv}{\sqrt{1+v^2}} = \int \frac{dx}{x}$

$\ln\left|v + \sqrt{x^2+1}\right| = \ln x + \ln c$

$\ln \frac{y}{x} + \sqrt{\frac{y^2}{x^2} + 1} = \ln cx$

$\ln \frac{y}{x} + \sqrt{\frac{y^2+x^2}{x^2}} = \ln cx$

$\frac{y}{x} + \frac{\sqrt{y^2+x^2}}{x} = cx$

$y + \sqrt{y^2+x^2} = cx^2$

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$y + \sqrt{x^2+y^2} = cx^2$

Date - 07.12.24

1/ $\frac{dy}{dx} = \frac{y-x+1}{y+x+5}$

$\frac{a_1}{a_2} = -1,\ \frac{b_1}{b_2} = 1$

$x = x+h,\ y = y+k$

$\frac{dy}{dx} = \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{y+k-(x+h)+1}{y+k+x+h+5}$

$= \frac{y-x-k-h+1}{y+x+k+h+5}$

$\frac{dy}{dx} = \frac{y-x}{y+x}$, homogenous

$k-h+1=0$

$k-h=-1$ — ①

$k+h+5=0$

$\frac{k+h=-5}{}$

$k=-3$

$h=-2$

$y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{vx-x}{vx+x} = \frac{x(v-1)}{x(v+1)}$

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$$x\frac{dv}{dx} = \frac{v-1}{v+1} - v = \frac{v-1-v^2-v}{v+1}$$

$$\int \frac{v+1}{v^2+1}\,dv = -\int \frac{dx}{x}$$

$$\int \frac{v\,dv}{v^2+1} + \int \frac{dv}{v^2+1} = -\ln x + c$$

$$\int \frac{dt}{2t} + \int \frac{dv}{v^2+1} = -\ln x + c \quad \left\{ \begin{array}{l} \text{Let } v^2+1 = t \\ 2v = \frac{dt}{dv} \\ v\,dv = \frac{dt}{2} \end{array} \right.$$

$$\frac{1}{2}\ln t + \tan^{-1} = -\ln x + c$$

$$\frac{1}{2}\ln(v^2+1) + \tan^{-1} v = \ln x + c$$

$$\ln \sqrt{\left(\frac{y^2}{x^2}+1\right)} + \tan^{-1} v = -\ln x + c$$

$$\ln \sqrt{\frac{y^2+x^2}{x}} + \ln x + \tan^{-1}\frac{y}{x} = c$$

$$\ln \frac{\sqrt{y^2+x^2}}{x} + \tan^{-1}\frac{y}{x} = c$$

$$\ln \sqrt{y^2+x^2} + \tan^{-1}\frac{y}{x} = c$$

$$\ln\sqrt{(y-k)^2 + (x-h)^2} + \tan^{-1} y k = c$$

$$= \ln\sqrt{(y-(-3))^2 + (x-(-2))^2} + \tan^{-1}\frac{y-(-3)}{x-(-2)} = c$$

$$= \ln\sqrt{(y+3)^2 + (x+2)^2} + \tan^{-1}\frac{y+3}{x+2} = c$$

10/ $(x-y)\,dy = (x+y+1)\,dx$

$$\frac{dy}{dx} = \frac{x+y+1}{x-y}$$

$$\frac{a_1}{a_2} = 1, \quad \frac{b_1}{b_2} = -1 \quad \left\{ \begin{array}{l} \frac{a_1}{a_2} = \frac{x}{x} = 1 \\ \frac{b_1}{b_2} = \frac{y}{-y} = -1 \end{array} \right.$$

$x = X+h, \quad y = Y+k$

$$\frac{dy}{dx} = \frac{dY}{dX}$$

$$\frac{dY}{dX} = \frac{X+h+Y+k+1}{X+h-Y-k}$$

$$= \frac{X+Y}{X-Y} \quad \text{homogenous}$$

$h-k = -1$

$h-k = 0$

$2h = -1$

$h = -\frac{1}{2}$

$k = -\frac{1}{2}$

$Y = vX$

$$\frac{dY}{dX} = v + X\frac{dv}{dX}$$

$$v + X\frac{dv}{dX} = \frac{X+Y}{X-Y} - v$$

$$X\frac{dv}{dX} = \frac{X+vX}{X-vX} - v$$

$$= \frac{X + \cancel{vX} - \cancel{vX} + v^2X}{X-vX}$$

$$= \frac{X(1+v^2)}{X(1-v)}$$

$$\int \frac{(1-v)\,dv}{1+v^2} = \int \frac{dX}{X}$$

$$\int \frac{dv}{1+v^2} - \int \frac{v\,dv}{1+v^2} = \int \frac{dX}{X}$$

$$\tan^{-1} v - \frac{1}{2}\ln(1+v^2) = \int \frac{dX}{X}$$

$$\tan^{-1}\frac{Y}{X} - \frac{1}{2}\ln\left(1+\frac{Y^2}{X^2}\right) = \frac{dX}{X}$$

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(10)

$\tan^{-1}\frac{y}{x} - \ln\frac{\sqrt{x^2+y^2}}{x} = \ln x + \ln c$

$\Rightarrow \tan^{-1}\frac{y}{x} - (\ln\sqrt{x^2+y^2} - \ln x) = \ln x + \ln c$

$\Rightarrow \tan^{-1}\frac{y}{x} - \ln\sqrt{x^2+y^2} + \ln x = \ln x + \ln c$

$\Rightarrow \tan^{-1}\frac{y}{x} - \ln\sqrt{x^2+y^2} = \ln c$

$\Rightarrow \tan^{-1}\frac{y}{x} = \ln c\sqrt{x^2+y^2}$

$\Rightarrow \tan^{-1}\frac{y-k}{x-h} = \ln c\sqrt{(x-h)^2+(y-k)^2}$

$\Rightarrow \tan^{-1}\frac{y+\frac{1}{2}}{x-\frac{1}{2}} = \ln c\sqrt{(x-\frac{1}{2})^2+(y-\frac{1}{2})^2}$

11) $(x-y-2)dx + (x-2y-3)dy = 0$

Ans → Given equation can be written as

$$\frac{dy}{dx} = \frac{x-y-2}{x-2y-3} \quad \text{--- (1)}$$

put $x = X+h$ and $y = Y+k$

So that $\frac{dy}{dx} = \frac{dY}{dX}$

Thus (1) is reduced to

$$\frac{dY}{dX} = \frac{(X-Y)+(h-k-2)}{(X-2Y)+(h-2k-3)} \quad \text{--- (2)}$$

Choose h, k such that

$h-k-2=0$ and $h-2k-3=0$

Subtracting we get

$2k+3-k-2=0 \Rightarrow k=-1$

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Again $h=2$

Thus (2) is reduced to

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Thus we have

$$v + x\frac{dv}{dx} = -\frac{x - vx}{x - 2vx} = -\frac{1-v}{1-2v}$$

$$\Rightarrow x\frac{dv}{dx} = -\left(\frac{1-v}{1-2v} + v\right)$$

$$= -\frac{1 - v + v - 2v^2}{1-2v} = -\frac{1-2v^2}{1-2v}$$

$$\Rightarrow \frac{1-2v}{1-2v^2}\,dv = \frac{dx}{x}$$

$$\Rightarrow \int \frac{1-2v}{1-2v^2}\,dv = -\int\frac{dx}{x}$$

$$\Rightarrow \int \frac{1}{1-2v^2}\,dv - \int\frac{2v\cdot dv}{1-2v^2} = -\int\frac{dx}{x}$$

$$\Rightarrow \int \frac{dv}{(1+\sqrt{2}v)(1-\sqrt{2}v)}$$

$$+\frac{1}{2}\int\frac{-4v}{1-2v^2}\,dv = -\int\frac{dx}{x}$$

$$\Rightarrow \frac{1}{2}\int\left\{\frac{1}{1+\sqrt{2}v} + \frac{1}{1-\sqrt{2}v}\right\}dv$$

$$\frac{1}{2}\int\left\{\frac{1}{1+\sqrt{2}v} + \frac{1}{1-\sqrt{2}v}\right\}dv = -\ln x + C$$

$$\Rightarrow \frac{1}{2}\cdot\frac{1}{\sqrt{2}}\ln\frac{1+\sqrt{2}v}{1-\sqrt{2}v} + \frac{1}{2}\ln(1-2v^2)$$

$$= -\ln x + C$$

$$\Rightarrow \frac{1}{2\sqrt{2}}\ln\frac{1+\sqrt{2}\frac{y}{x}}{1-\sqrt{2}\frac{y}{x}} + \frac{1}{2}\ln\left(1-2\frac{y^2}{x^2}\right)$$

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$= -\ln X + c$

(1) $\Rightarrow \frac{1}{2\sqrt{2}} \ln \frac{X+\sqrt{2}Y}{X-\sqrt{2}Y} + \frac{1}{2} \ln \frac{X^2-2Y^2}{X^2} + \ln X = c$

$\Rightarrow \frac{1}{2\sqrt{2}} \ln\left(\frac{X+\sqrt{2}Y}{X-\sqrt{2}Y}\right) + \ln\left(\sqrt{\frac{X^2-2Y^2}{X^2}} \cdot X\right) = c$

$\Rightarrow \frac{1}{2\sqrt{2}} \ln \frac{X+\sqrt{2}Y}{X-\sqrt{2}Y} + \ln\left(\sqrt{X^2-2Y^2}\right) = c$

$\left[\because X = x-h = x-1, \quad Y = y-k = y+1\right]$

$\Rightarrow \frac{1}{2\sqrt{2}} \ln\left[\frac{(x-1)+\sqrt{2}(y+1)}{(x-1)-\sqrt{2}(y+1)}\right] + \ln\sqrt{(x-1)^2-2(y+1)^2} = c$

This is the required solution.

(12) $\frac{dx}{dy} = \frac{3x-7y+7}{3y-7x-3}$

Ans $\Rightarrow$ Given eqn. can be written as

$\cancel{\frac{dy}{dx} = \frac{3y-7y}{3x-7x}}$

$\frac{dy}{dx} = \frac{3y-7x-3}{3x-7y+7}$

proceeding as in Q. 11 we get

$\frac{dY}{dX} = \frac{3Y-7X}{3X-7Y}$ ——(1)

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[where $3k-7h-3=0$

and $3h-7k+7=0 \Rightarrow 7h-3k+3=0$

and $3h-7k+7=0$

Solving $\dfrac{h}{-21+21}=\dfrac{k}{9-49}=\dfrac{1}{-49+9}$

$\Rightarrow \dfrac{h}{0}=\dfrac{k}{-40}=\dfrac{1}{-40}$

$\Rightarrow \boxed{h=0,\ k=1}$]

Again putting $y=vx$ and proceeding again

we get $v+x\dfrac{dv}{dx}=\dfrac{3v-7}{3-7v}$

$\Rightarrow x\dfrac{dv}{dx}=\dfrac{3v-7}{3-7v}-v$

$=\dfrac{3v-7-3v+7v^2}{3-7v}$

$=\dfrac{-7+7v^2}{3-7v}=-\dfrac{7-7v^2}{3-7v}$

$\Rightarrow \dfrac{3-7v}{7-7v^2}dv=-\dfrac{dx}{x}$

$\Rightarrow \dfrac{3}{7}\displaystyle\int\dfrac{dv}{1-v^2}-\int\dfrac{v}{1-v^2}dv=-\int\dfrac{dx}{x}$

$\Rightarrow \dfrac{3}{14}\displaystyle\int\left\{\dfrac{1}{1+v}+\dfrac{1}{1-v}\right\}dv+\dfrac{1}{2}\int\dfrac{-2v}{1-v^2}dv=-\int\dfrac{dx}{x}$

$\Rightarrow \dfrac{3}{14}\ln\left(\dfrac{1+v}{1-v}\right)+\dfrac{1}{2}\ln(1-v^2)=-\ln x+c$

$\Rightarrow \dfrac{3}{14}\ln\left(\dfrac{x+y}{x-y}\right)+\ln\dfrac{\sqrt{x^2-y^2}}{x}+\ln x=c$

$\Rightarrow \dfrac{3}{14}\ln\left(\dfrac{x+y}{x-y}\right)+\ln\sqrt{x^2-y^2}=c$

$\Rightarrow \dfrac{3}{14}\ln\left(\dfrac{x+y-1}{x-y+1}\right)+\ln\sqrt{x^2-(y-1)^2}=c$

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$\left[\because x = x-h = x,\ y = y-k = y-1\right]$

This is the required solution.

13) $(2x+y+1)\,dx + (4x+2y-1)\,dy = 0$

⇒ Given eqn can be written as

$$\frac{dy}{dx} = -\frac{2x+y+1}{4x+2y-1}$$

$$= -\frac{(2x+y)+1}{2(2x+y)-1} \quad \text{——(1)}$$

put

$2x+y = z$

Then $2+\frac{dy}{dx} = \frac{dz}{dx}$ or $\frac{dy}{dx} = \frac{dz}{dx} - 2$

Thus (1) reduces to $\frac{dz}{dx} - 2 = -\frac{z+1}{2z-1}$

$$\Rightarrow \frac{dz}{dx} = 2 - \frac{z+1}{2z-1} = \frac{4z-2-z-1}{2z-1} = \frac{3z-3}{2z-1}$$

$$\Rightarrow \frac{2z-1}{3(z-1)}\,dz = dx \Rightarrow \frac{2z-1}{z-1}\,dz = 3\,dx$$

$$\Rightarrow \frac{2(z-1)+1}{(z-1)}\,dz = 3\,dx$$

$$\Rightarrow \int\left\{2+\frac{1}{z-1}\right\}dz = 3\int dx$$

$$\Rightarrow 2z + \ln(z-1) = 3x + c$$

$$\Rightarrow 2(2x+y) + \ln(2x+y-1) = 3x + c$$

$$\Rightarrow (x+2y) + \ln(2x+y-1) = c$$

This is the required solution.

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14) $(2x-3y-5)\frac{dy}{dx}+3x+2y-5=0$

$\Rightarrow$ Given equation can be witten as

$$\frac{dy}{dx} = -\frac{3x+2y-5}{2x+3y-5} \quad \text{---} (1)$$

put $x = X+h$ and $y = Y+k$

so that $\frac{dy}{dx} = \frac{dY}{dX}$

Thus (1) become

$$\frac{dY}{dX} = -\frac{(3X+2Y)+(3h+2k-5)}{(2X+3Y)+(2h+3k-5)}$$

[Choose h and k such that

$3h+2k-5=0$ and $2h+3k-5=0$

Solving we get

$$\frac{h}{-10+15} = \frac{k}{-10+15} = \frac{1}{9-4}$$

$\Rightarrow \frac{h}{5} = \frac{k}{5} = \frac{1}{5} \Rightarrow h=k=1$

Thus we have $\frac{dY}{dX} = -\frac{3X+2Y}{2X+3Y}$

put $Y = vX$

Then $\frac{dY}{dX} = v + X\frac{dv}{dX}$

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Thus we have

$$v + x\frac{dv}{dx} = -\frac{3x+2vx}{2x+3vx} = -\frac{3+2v}{2+3v}$$

$$\Rightarrow x\frac{dv}{dx} = -\frac{3+2v}{2+3v} - v$$

$$= -\left\{\frac{3+2v+3v^2}{2+3v}\right\}$$

$$= -\frac{3+4v+3v^2}{2+3v}$$

$$\Rightarrow \frac{2+3v}{3+4v+3v^2}\,dv = -\int\frac{dx}{x}$$

$$\Rightarrow \int\frac{2+3v}{3+4v+3v^2}\,dv = -\int\frac{dx}{x}$$

$$\Rightarrow \frac{1}{2}\int\frac{6v+4}{3v^2+4v+3}\,dv = -\int\frac{dx}{x}$$

$$\Rightarrow \frac{1}{2}\ln(3v^2+4v+3) = -\ln x + \ln c$$

$$\Rightarrow \ln\sqrt{3v^2+4v+3} = \ln\frac{c}{x}$$

$$\Rightarrow \sqrt{3v^2+4v+3} = \frac{c}{x}$$

$$\Rightarrow \sqrt{3\frac{y^2}{x^2}+4\frac{y}{x}+3} = \frac{c}{x}$$

$$\Rightarrow \sqrt{\frac{3y^2+4xy+3x^2}{x}} = \frac{c}{x}$$

$$\Rightarrow \sqrt{3y^2+4xy+3x^2} = c$$

Squaring both both sides we get

$$3y^2+4xy+3x^2 = c^2 = C$$

∴ The solution is

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$3(y-1)^2+4(x-1)(y-1)+3(x-1)^2=c$

(15) $(4x+6y-5)dx-(2x+3y+4)dy=0$

Ans:- given equation can be written as

$$\frac{dy}{dx}=\frac{4x+6y+5}{2x+3y+4}=\frac{2(2x+3y)+5}{(2x+3y)+4} \quad \text{——(1)}$$

put $2x+3y=z$

Then $2+3\frac{dy}{dx}=\frac{dz}{dx}$ or $\frac{dy}{dx}=\frac{\frac{dz}{dx}-2}{3}$

putting these in (1) we get

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$$\frac{\frac{dz}{dx}-2}{3}=\frac{2z+5}{z+4}$$

$$\Rightarrow \frac{dz}{dx}=\frac{6z+15}{z+4}+2$$

$$=\frac{6z+15+2z+8}{z+4}=\frac{8z+23}{z+4}$$

$$\Rightarrow \frac{z+4}{8z+23}dz=dx$$

$$\Rightarrow \int\frac{z+4}{8z+23}dz=\int dx$$

$$\Rightarrow \frac{1}{8}\int\frac{(8z+23)+9}{8z+23}dz=\int dx$$

$$\Rightarrow \int\left\{1+\frac{9}{8z+23}\right\}dz=8\int dx$$

$$\Rightarrow z+\frac{9}{8}\ln(8z+23)=8x+c$$

$$\Rightarrow 2x+3y+\frac{9}{8}\ln 8(2x+3y)+23)=8x+c$$

$$\Rightarrow 2x + 3y + \frac{9}{8}\ln(16x + 24y + 23) = 8x + c$$

$$\Rightarrow 3y - 6x + \frac{9}{8}\ln(16x + 24y + 23) = c$$

This is the required solution.

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