VECTORS

(1) When two vectors $\vec{a}$ and $\vec{b}$ are parallel then their cross product (×) is zero. i.e $\vec{a} \times \vec{b} = 0$ when $\vec{a} \parallel \vec{b}$.

(2) When two vectors $\vec{a}$ and $\vec{b}$ are perpendicular then their dot product (·) is zero. i.e $\vec{a} \cdot \vec{b} = 0$ when $\vec{a} \perp \vec{b}$.

VECTORS CHAPTER-12 Date-20.02.24

Important formulae:

1/ If $P = (x_1, y_1, z_1)$ and $Q = (x_2, y_2, z_2)$
then
$\vec{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$
where $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors along x-axis y-axis and z axis.

2/ Magnitude of a vector

$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ is defined by
$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

3/ Unit vector along the vector $\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$

4/ (i) The dot product of two vectors $\vec{a}$ and $\vec{b}$ defined by $\vec{a} \cdot \vec{b} = ab\cos\theta$
where $a = |\vec{a}|$, $b = |\vec{b}|$ and θ is angle between $\vec{a}$ and $\vec{b}$.

(ii) $\theta = \cos^{-1}\left(\dfrac{\vec{a} \cdot \vec{b}}{ab}\right)$

(iii) Two vectors $\vec{a}$ and $\vec{b}$ are orthogonal if $\vec{a} \cdot \vec{b} = 0$

5/ (i) scalar projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

(ii) scalar projection of $\vec{b}$ on $\vec{a} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$

(iii) vector projection of

$$\vec{a} \text{ on } \vec{b} = \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\right)\frac{\vec{b}}{|\vec{b}|}$$

(iv) vector projection of

$$\vec{b} \text{ on } \vec{a} = \left(\frac{\vec{b}\cdot\vec{a}}{|\vec{a}|}\right)\frac{\vec{a}}{|\vec{a}|}$$

6/ <u>properties of dot product.</u>

(i) Dot product is commutative i.e

$$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}.$$

(ii) Dot product is distributive over vector addition i.e $\vec{a}\cdot(\vec{b}+\vec{c}) = \vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}$

(iii) If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k} \text{ then}$$

$$\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$$

7/ vector product or cross product of two vectors $\vec{a}$ and $\vec{b}$ is defined by $\vec{a}\times\vec{b} = ab\sin\theta\,\hat{n}$, where θ is the angle between $\vec{a}$ and $\vec{b}$ and $\hat{n}$ is the unit vector perpendicular

$\vec{a}$ and $\vec{b}$ such that $\vec{a}, \vec{b}, \hat{n}$ are in right handed orientation.

8/ properties of vector product:

(i) Area of a parallelogram whose adjacent sides are represented by the vectors $\vec{a}, \vec{b}$ is $|\vec{a} \times \vec{b}|$

(ii) Area of the triangle whose two sides are $\vec{a}$ and $\vec{b}$ is $\frac{1}{2}|\vec{a} \times \vec{b}|$.

(iii) $\vec{a} \times \vec{a} = \vec{0}$.

(iv) $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = \vec{0}$

$\hat{i} \times \hat{j} = \hat{k}, \hat{j} \times \hat{k} = \hat{i}, \hat{k} \times \hat{i} = \hat{j}$

(v) If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

vector product is not commutative i.e $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$.

(vi) vector product is distributive over vector addition i.e.

$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$.

(vii) $\sin\theta = \dfrac{|\vec{a} \times \vec{b}|}{ab}$

(viii) $\hat{n} = \dfrac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$

9/ The scalar triple of three vectors $\vec{a}, \vec{b}, \vec{c}$ is defined by

$$[\vec{a}\ \vec{b}\ \vec{c}] = \vec{a} \cdot (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \cdot \vec{c}$$

10/ properties of scalar triple product:

(i) In scalar triple product dot and cross can be interchanged.

(ii) If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

then $\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

(iii) The scalar triple product represents the volume of the parallelopiped formed by the three vectors.

(iv) Three vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar if $[\vec{a}, \vec{b}, \vec{c}] = 0$

Exercise - 16 (a)

1/ (i) Given $\vec{a}$ =
Two pair of vectors have same direction

(ii) $\frac{2}{\alpha} = \frac{3}{-1} = \frac{-6}{2}$

$\frac{2}{\alpha} = \frac{3}{-1}$

$\alpha = -\frac{2}{3}$

(iii) $\vec{A} = 3\hat{i} + \hat{k}$

$\vec{B} = 2\hat{i} + \hat{j} - \hat{k}$

$\vec{BA} = \hat{i} - \hat{j} + 2\hat{k}$

(iv) $|K\vec{a}| = 1$

$K = \frac{\pm 1}{|\vec{a}|}$

(v) $\vec{OP} = (1, 0, -2)$

$\vec{OQ} = (3, -2, 0)$

$\vec{PQ} = \vec{OQ} - \vec{OP} = (3, -2, 0) - (1, 0, -2)$

$= 2\hat{i} - 2\hat{j} + 2\hat{k}$ $\quad |\vec{PQ}| = \sqrt{(2)^2 + (-2)^2 + (2)^2}$

$= \sqrt{12} = 2\sqrt{3}$

$\cos\alpha = \frac{\cancel{2}}{\cancel{2}\sqrt{3}}$, $\cos\beta = \frac{-\cancel{2}}{\cancel{2}\sqrt{3}}$, $\cos\gamma = \frac{\cancel{2}}{\cancel{2}\sqrt{3}}$

$\therefore$ direction cosines $\left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

2/ i $\vec{a} - \vec{a} = \vec{a}$

(ii) The vector $\vec{0}$ has arbitrary direction.

(iii) All unit vector are equal in magnitude.

(iv) $|\vec{a}| = |\vec{b}| \Rightarrow \vec{a} = \vec{b}$ But $\vec{a} = 2\hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{b} = -2\hat{i} - 2\hat{j} - 2\hat{k}$

Not always true

(v) Substraction of vector is not commutative

$\vec{a} - \vec{b} \neq \vec{b} - \vec{a}$

3/(i) $\vec{a} = (2,1) = 2\hat{i} + \hat{j}$

$\vec{b} = (-1, 0) = -\hat{i}$

Date - 11. 02. 24

3/(i) $\vec{a} = (2,1) = 2\hat{i} + \hat{j}$

$\vec{b} = (-1,0) = -\hat{i}$

$3\vec{a} + 2\vec{b} = 6\hat{i} + 3\hat{j} - 2\hat{i}$

$= 4\hat{i} + 3\hat{j}$

(ii) $\vec{a} = (1,1,1) = \hat{i} + \hat{j} + \hat{k}$

$\vec{b} = (-1, 3, 0) = -\hat{i} + 3\hat{j}$

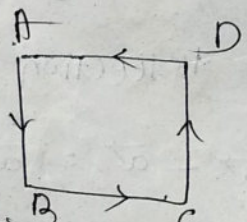
$\vec{c} = (2, 0, 2) = 2\hat{i} + 2\hat{k}$

$\vec{a} + 2\vec{b} - \frac{1}{2}\vec{c}$

$= \hat{i} + \hat{j} + \hat{k} - 2\hat{i} + 6\hat{j} - \hat{i} - \hat{k}$

$= -2\hat{i} + 7\hat{j}$

4/ $\vec{AB} = -\vec{CD}$



$\vec{BC} = -\vec{DA}$

$\vec{AB} + \vec{BC} + \vec{CD} + \vec{DA}$

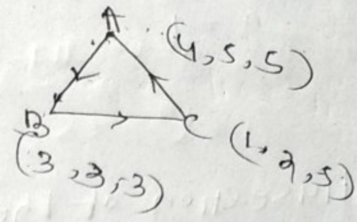
$= -\vec{CD} - \vec{DA} + \vec{CD} + \vec{DA}$

$= 0$

5/(i) $\vec{AB} = -\hat{i} - 2\hat{j} - 2\hat{k}$

$\vec{BC} = -2\hat{i} - \hat{j} + 2\hat{k}$

$\vec{CA} = 3\hat{i} + 3\hat{j}$



length of (AB)

$= |\vec{AB}| = \sqrt{(-1)^2 + (-2)^2 + (-2)^2}$

$= \sqrt{1+4+4} = 3$

$|\vec{BC}| = \sqrt{(-2)^2 + (-1)^2 + (2)^2}$

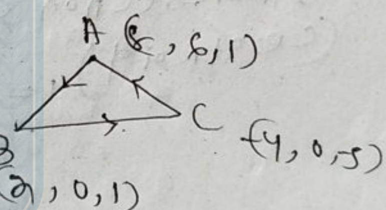
$= \sqrt{4+1+4} = 3$

$|\vec{CA}| = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

(ii) $\vec{AB} = -6\hat{i} - 6\hat{j}$

$\vec{BC} = -6\hat{i} - 6\hat{k}$

$\vec{CA} = 12\hat{i} + 6\hat{j} + 6\hat{k}$



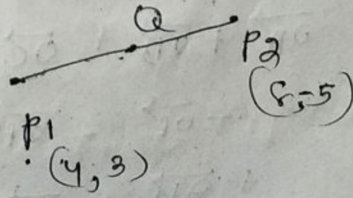
$|\vec{AB}| = \sqrt{(-6)^2 + (-6)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$

$|\vec{BC}| = \sqrt{(-6)^2 + (-6)^2} = \sqrt{72} = 6\sqrt{2}$

$|\vec{CA}| = \sqrt{(12)^2 + (6)^2 + (6)^2} = \sqrt{144+36+36}$

$= \sqrt{216} = 6\sqrt{6}$

6/ $\boxed{\text{mid point form} = \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}}$

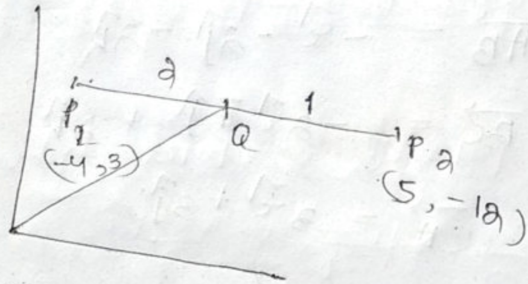


mid point of $\vec{P_1P_2} = \left(\frac{8+4}{2}, \frac{-5+3}{2}\right)$

$= (6, -1)$

(7)

$m = 2$
$n = 1$



Trisection of the vector $\overrightarrow{P_1P_2} =$

$$\frac{mx_1 + nx_2}{m+n}, \frac{my_1 + ny_2}{m+n}$$

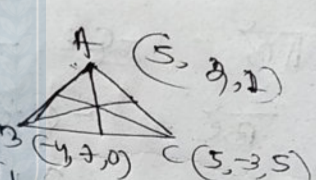
$$= \frac{2(-4) + 1(5)}{2+1}, \frac{2(3) + 1(-12)}{2+1}$$

$$= \frac{-8+5}{3}, \frac{6-12}{3}$$

$$= (-1, -2)$$

$$= -\hat{i} - 2\hat{j}$$

8/ Intersection of medians (centroid) $= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$



$$= \left(\frac{5 - 4 + 5}{3}, \frac{2 + 7 - 3}{3}, \frac{1 + 0 + 5}{3}\right)$$

$$= (2, 2, 2)$$

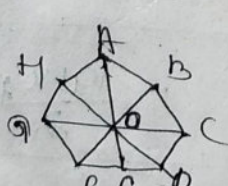
$$= 2\hat{i} + 2\hat{j} + 2\hat{k}$$

9/ $\overrightarrow{OA} = -\overrightarrow{OE}$

$\overrightarrow{OB} = -\overrightarrow{OF}$

$\overrightarrow{OC} = -\overrightarrow{OG}$

$\overrightarrow{OD} = -\overrightarrow{OH}$



$\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} + \overrightarrow{OE} + \overrightarrow{OF} + \overrightarrow{OG} + \overrightarrow{OH}$

$= -\overrightarrow{OE} - \overrightarrow{OF} - \overrightarrow{OG} - \overrightarrow{OH} + \overrightarrow{OE} + \overrightarrow{OF} + \overrightarrow{OG} + \overrightarrow{OH} = \vec{0}$

10/ $\vec{OA}+\vec{AB}=\vec{OB}$

$\vec{OB}+\vec{BC}=\vec{OC}$

$\vec{OC}+\vec{CD}=\vec{OD}$

$\vec{OA}+\vec{AB}+\vec{BC}+\vec{CD}+\vec{DO}$

$=\vec{OB}+\vec{BC}+\vec{CD}+\vec{DO}$

$=\vec{OC}+\vec{CD}+\vec{DO}$

$=\vec{OD}+\vec{DO}=\vec{OD}-\vec{OD}$

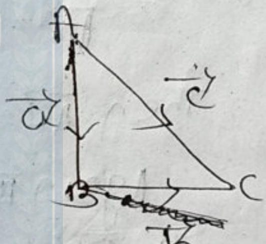
$=\vec{0}$

11/(i) (a) (i) $|\vec{a}+\vec{b}| \le |\vec{a}|+|\vec{b}|$

In $\triangle ABC$ $\vec{AB}=\vec{a}$

$\vec{BC}=\vec{b}$

$\vec{AC}=\vec{c}$

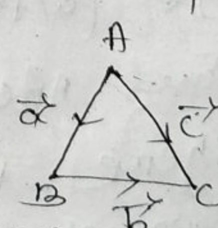


We know that $\vec{AB}+\vec{BC}>\vec{AC}$

$|\vec{a}|+|\vec{b}|>\vec{c}$

$|\vec{a}|+|\vec{b}|>|\vec{a}+\vec{b}|$

(ii) $|\vec{a}-\vec{b}| \ge |\vec{a}|-|\vec{b}|$



In $\triangle ABC = \vec{AB}=\vec{a}, \vec{BC}=\vec{b}, \vec{AC}=\vec{c}$

Pf:-

$|\vec{AB}-\vec{BC}|<|\vec{AC}|$

$|\vec{a}-\vec{b}|<|\vec{c}|$

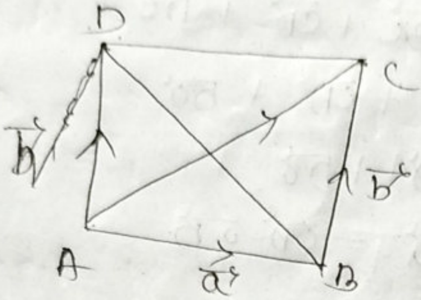
$|\vec{a}-\vec{b}|<|\vec{a}|+|\vec{b}|$

$|\vec{a}-\vec{b}| \ge |\vec{a}|-|\vec{b}|$

(b) $|\vec{a}+\vec{b}| = |\vec{a}| - |\vec{b}|$

The geometrical significance of the rel? is

(b)



Let ABCD be a parallelogram where

$\vec{AB} = \vec{a}, \vec{BC} = \vec{b}$

Then $\vec{AC} = \vec{a}+\vec{b}$

Again $\vec{AD} = \vec{b}$

So $\vec{DB} = \vec{a}-\vec{b}$

Now $AC = |\vec{AC}| = |\vec{a}+\vec{b}|$

$DB = |\vec{DB}| = |\vec{a}-\vec{b}|$

If $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$ then $AC = DB$

Thus ABCD is a parallelogram where two diagonals are equal.

Hence ABCD must be a ~~rectangual~~ rectangle with adjacent vectors $\vec{a}$ and $\vec{b}$

2/(i) $P(-1,3) = -\hat{i} + 3\hat{j}$

$Q(1,2) = \hat{i} + 2\hat{j}$

$\overrightarrow{PQ} = 2\hat{i} - \hat{j}$

$|\overrightarrow{PQ}| = \sqrt{(2)^2 + (-1)^2} = \sqrt{4+1} = \sqrt{5}$

(ii) $P(-1,-2) = -\hat{i} - 2\hat{j}$

$Q = (-5,-6) = -5\hat{i} - 6\hat{j}$

$\overrightarrow{PQ} = -4\hat{i} - 4\hat{j}$

$|\overrightarrow{PQ}| = \sqrt{(-4)^2 + (-4)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$

(iii) $P(1,4,-3) = \hat{i} + 4\hat{j} - 3\hat{k}$

$Q(2,-2,-1) = \hat{i} - 2\hat{j} - \hat{k}$

$\overrightarrow{PQ} = \hat{i} - 6\hat{j} + 2\hat{k}$

$|\overrightarrow{PQ}| = \sqrt{(1)^2 + (-6)^2 + (2)^2} = \cancel{\sqrt{36+1+4}}$

$= \sqrt{1+36+4}$

$= \sqrt{41}$

(13)

(i) $P(2,-1,-1), Q(-1,-3,2)$

Ans → (i) $\overrightarrow{PQ} = (-1-2)\hat{i} + (-3+1)\hat{j} + (2+1)\hat{k}$

$= -3\hat{i} - 2\hat{j} + 3\hat{k}$

$|\overrightarrow{PQ}| = \sqrt{9+4+9} = \sqrt{22}$

direction cosines ~~D.Cs~~ of $\overrightarrow{PQ}$ are $\frac{-3}{\sqrt{22}}, \frac{-2}{\sqrt{22}}, \frac{3}{\sqrt{22}}$

(ii) $P(3,-1,7)$ $Q(4,-3,-1)$

Ans → $\overrightarrow{PQ} = (4-3)\hat{i} + (-3+1)\hat{j} + (-1-7)\hat{k}$

$= \hat{i} - 2\hat{j} - 8\hat{k}$

$|\overrightarrow{PQ}| = \sqrt{1+4+64} = \sqrt{69}$

D.cs of $\overrightarrow{PQ}$ are $\frac{1}{\sqrt{69}}, \frac{-2}{\sqrt{69}}, \frac{-8}{\sqrt{69}}$

14/ Ans → $\vec{a} - \vec{b} + 2\vec{c} = (2,-2,1) - (2,3,6) + 2(-1,0,2)$

$= (2,-2,1) + (-2,-3,-6) + (-2,0,4)$

$= (2-2, -2, -2-3+0, 1-6+4)$

$= (-2, -5, -1)$

Magnitude of $\vec{a} - \vec{b} + 2\vec{c} = \sqrt{4+25+1} = \sqrt{30}$

D.cs of $\vec{a} - \vec{b} + 2\vec{c}$ are $\frac{-2}{\sqrt{30}}, \frac{-5}{\sqrt{30}}, \frac{-1}{\sqrt{30}}$

15(i)/ $\vec{a} = 5\hat{i} + 12\hat{j}$

$|\vec{a}| = \sqrt{(5)^2 + (12)^2}$

$= \sqrt{25 + 144}$

$= \sqrt{169}$

$= 13$

The unit vector along $\vec{a} = \frac{5}{13}\hat{i}, \frac{12}{13}\hat{j}$,

(ii) $\vec{a} = 2\hat{i} + \hat{j}$

$|\vec{a}| = \sqrt{(2)^2 + (1)^2}$

$= \sqrt{4+1} = \sqrt{5}$

The unit vector along $\vec{a} = \frac{2}{\sqrt{5}}\hat{i}, \frac{1}{\sqrt{5}}\hat{j}$,

(iii) $\vec{a} = 3\hat{i} + 6\hat{j} - \hat{k}$

$|\vec{a}| = \sqrt{(3)^2 + (6)^2 + (-1)^2}$

$= \sqrt{9 + 36 + 1}$

$= \sqrt{46}$

The unit vector along $\frac{3}{\sqrt{46}}\hat{i}, \frac{6}{\sqrt{46}}\hat{j}, \frac{-1}{\sqrt{46}}$,

(iv) $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$

$|\vec{a}| = \sqrt{(3)^2 + (1)^2 + (-2)^2}$

$= \sqrt{9+1+4} = \sqrt{14}$

The unit vector along $\frac{3}{\sqrt{14}}\hat{i}, \frac{1}{\sqrt{14}}\hat{j}$, $\frac{-2}{\sqrt{14}}\hat{k}$.

16/ Ans→ $\vec{r_1} - \vec{r_2} = \hat{i} + 2\hat{j} + \hat{k} - 3\hat{i} - \hat{j} + 5\hat{k}$

$= -2\hat{i} + \hat{j} + 6\hat{k}$

$|\vec{r_1} - \vec{r_2}| = \sqrt{4+1+36} = \sqrt{41}$

unit vector along $\vec{r_1} - \vec{r_2}$

$= \frac{-2\hat{i} + \hat{j} + 6\hat{k}}{\sqrt{41}} = \frac{-2}{\sqrt{41}}\hat{i} + \frac{1}{\sqrt{41}}\hat{j} + \frac{6}{\sqrt{41}}\hat{k}$

17/ Ans→ $\vec{a} + \vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k} + \hat{i} + 2\hat{j} + 3\hat{k}$

$= 3\hat{i} + 6\hat{j} - 2\hat{k}$

$|\vec{a} + \vec{b}| = \sqrt{9+36+4} = 7$

Unit vector parallel to $\vec{a} + \vec{b} = \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{7}$

D.Cs are $\frac{3}{7}, \frac{6}{7}, \frac{-2}{7}$

18/ Let $\hat{a}, \hat{b}, \hat{c}$ are 3 unit vectors

such that $\hat{a} + \hat{b} = \hat{c}$

$(\vec{a}+\vec{b})^2 = |\vec{c}|^2$

$|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b} = 1$

$1+1 + 2\vec{a}\cdot\vec{b} = 1$

$2\vec{a}\cdot\vec{b} = 1-2 = -1$

$(|\vec{a}-\vec{b}|)^2 = |\vec{a}|^2 + |\vec{b}| - 2\vec{a}\cdot\vec{b}$

$= 1+1-(-1) = 1+1+1 = 3$

$|\vec{a}-\vec{b}| = \sqrt{3}$

(19)

$\vec{A} = 4\hat{i} + 3\hat{j} - \hat{k}$

$\vec{B} = 5\hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{C} = 2\hat{i} - 2\hat{j} - 3\hat{k}$

$\vec{D} = 4\hat{i} - 4\hat{j} + 3\hat{k}$

$\vec{AB} = \hat{i} - \hat{j} + 3\hat{k}$

$\vec{CD} = 2\hat{i} - 2\hat{i} + 6\hat{k}$

$= 2(\hat{i} - \hat{j} + 3\hat{k})$

$\vec{CD} = 2\,\vec{AB}$

(20) (i) $\vec{OA} = 2\hat{i} + 6\hat{j} + 3\hat{k}$

$\vec{OB} = \hat{i} + 2\hat{j} + 7\hat{k}$

$\vec{OC} = 3\hat{i} + 10\hat{j} - \hat{k}$

$\vec{AB} = -\hat{i} - 4\hat{j} + 4\hat{k}$

$\vec{AC} = \hat{i} + 4\hat{j} - 4\hat{k}$

$\vec{AB} = -\vec{AC}$

So, $\vec{OA}$, $\vec{OB}$, $\vec{OC}$ are wllecinear.

(ii)

Given that $P = (2,-1,3)$

$Q = (3,-5,1)$ and $R = (-1,11,9)$

Then $\overrightarrow{PQ} = (3-2)\hat{i} + (-5+1)\hat{j} + (1-3)\hat{k}$

$= \hat{i} - 4\hat{j} - 2\hat{k}$

$\overrightarrow{QR} = (-1-3)\hat{i} + (11+5)\hat{j} + (9-1)\hat{k}$

$= -4\hat{i} + 16\hat{j} + 8\hat{k}$

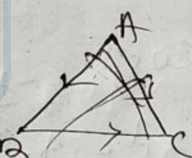
$= -4(\hat{i} - 4\hat{j} - 2\hat{k}) = -4\overrightarrow{PQ}$

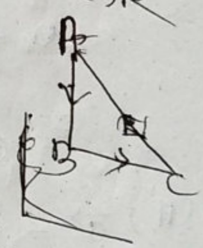
$\Rightarrow$ $\overrightarrow{PQ}$ and $\overrightarrow{QR}$ are parallel vectors having Q as common point. Hence the point P, Q, R are collinear (proved)

12/ $\overrightarrow{AB} = 2\hat{i} - \hat{j} + \hat{k}$

$\overrightarrow{BC} = \hat{i} - 3\hat{j} - 5\hat{k}$

$\overrightarrow{CA} = 3\hat{i} - 4\hat{j} - 4\hat{k}$





$|\overrightarrow{AB}| = \sqrt{(2)^2 + (-1)^2 + (1)^2} = \sqrt{6}$

$|\overrightarrow{BC}| = \sqrt{1+9+25} = \sqrt{35}$

$|\overrightarrow{CA}| = \sqrt{9+16+16} = \sqrt{41}$

$|\overrightarrow{CA}| = \sqrt{|\overrightarrow{BC}|^2 + |\overrightarrow{AB}|^2}$

$= \sqrt{(\sqrt{35})^2 + (\sqrt{6})^2}$

$= \sqrt{35+6} = \sqrt{41}$

So the following vectors are sides of a right angle triangle.